Counting Words:
Type-rich populations, samples, and statistical models

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Why we need the population

There are two reasons why we want to construct a model of the type population distribution:

- Population distribution is interesting by itself, for theoretical reasons or in NLP applications
- We know how to simulate sampling from population → once we have a population model, we can obtain estimates of $V(N)$, $V_1(N)$ and similar quantities for arbitrary sample sizes N

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A third reason:

- The bell-bottom shape of the observed Zipf ranking does not fit Zipf's law (type frequencies must be integers!)
- It is more natural to characterize occurrence *probabilities* (for which there is no such restriction) by Zipf's law

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- It is customary (and convenient) to arrange types in order of decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
- NB: this is usually *not* the same ordering as in the observed Zipf ranking (we will see examples of this later)

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  - e.g. psycholinguistic model of a human speaker
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- analogous interpretations for other linguistic and non-linguistic phenomena

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- But we cannot reliably estimate thousands or millions of π_k's from any finite sample (just think of all the unseen types that do not occur in the sample)

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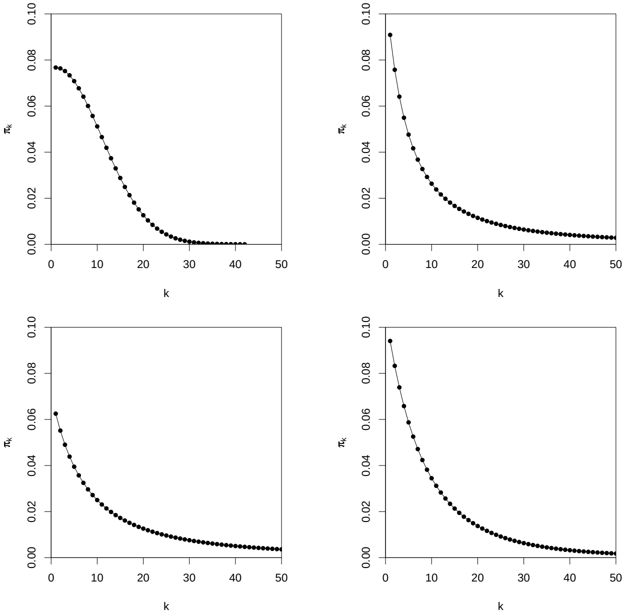
- These parameters can be estimated with relative ease

Examples of population models



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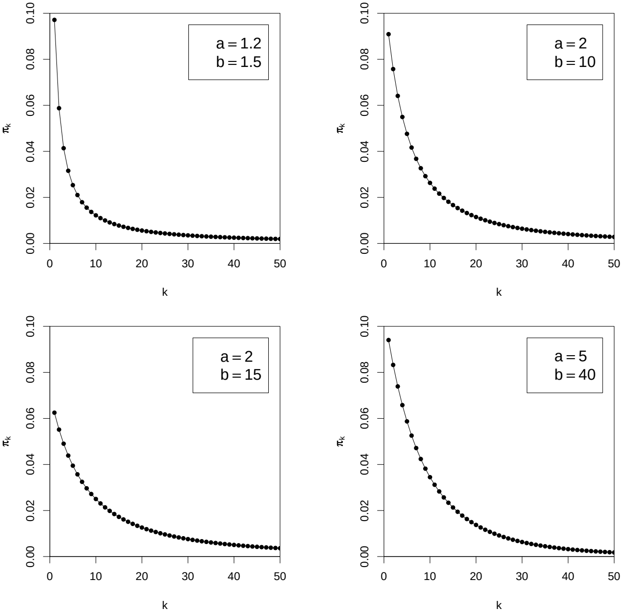

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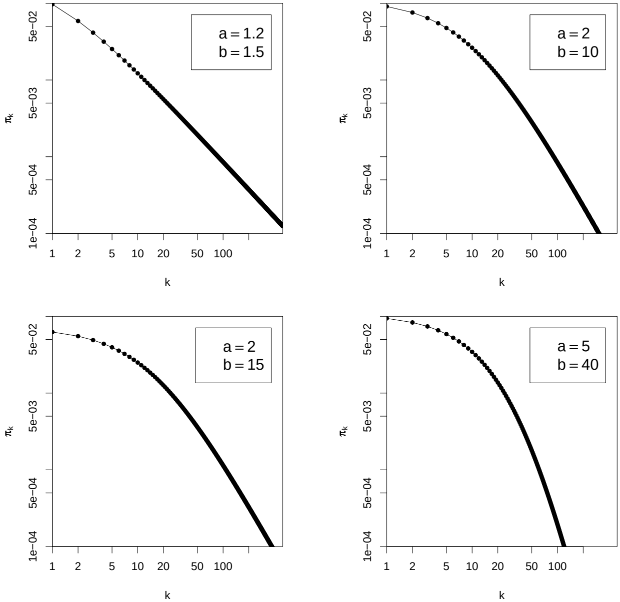

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- Alternative: finite (but often very large) number of types in the population
- We call this the **population vocabulary size** S (and write $S = \infty$ for an infinite type population)

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Abbreviations: **ZM** for **Zipf-Mandelbrot** model,
and **fZM** for **finite Zipf-Mandelbrot** model

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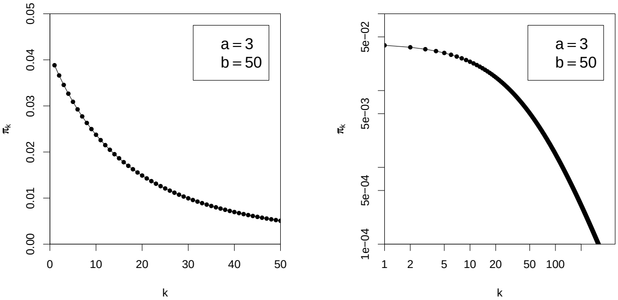
  - it will also allow us to estimate the model parameters

Outline

Sampling from a population model

Assume we believe that the population we are interested in can be described by a Zipf-Mandelbrot model:



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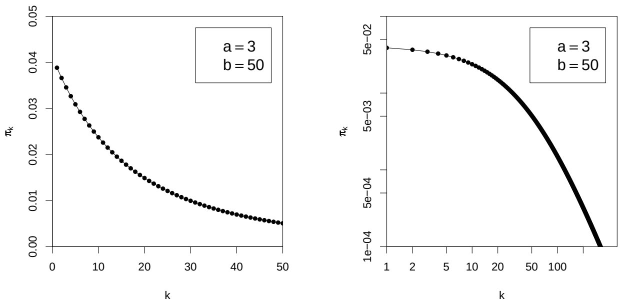

Use computer simulation to sample from this model:

- Draw N tokens from the population such that in each step, type w_k has probability π_k to be picked

Sampling from a population model

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#5:	24	81	54	11	8	61	1	31	35	...
#6:	3	65	9	165	5	42	16	20	7	...
#7:	10	21	11	60	164	54	18	16	203	...
#8:	11	7	147	5	24	19	15	85	37	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

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Here are some results for samples of size $N = 1000$ ...

Samples: type frequency list & spectrum

rank r	f_r	type k
1	37	6
2	36	1
3	33	3
4	31	7
5	31	10
6	30	5
7	28	12
8	27	2
9	24	4
10	24	16
11	23	8
12	22	14
⋮	⋮	⋮

m	V_m
1	83
2	22
3	20
4	12
5	10
6	5
7	5
8	3
9	3
10	3
⋮	⋮

sample #1

Samples: type frequency list & spectrum

rank r	f_r	type k
1	39	2
2	34	3
3	30	5
4	29	10
5	28	8
6	26	1
7	25	13
8	24	7
9	23	6
10	23	11
11	20	4
12	19	17
$\vdots$	$\vdots$	$\vdots$

m	V_m
1	76
2	27
3	17
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6	5
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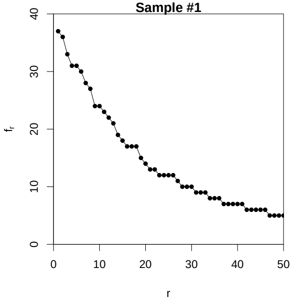
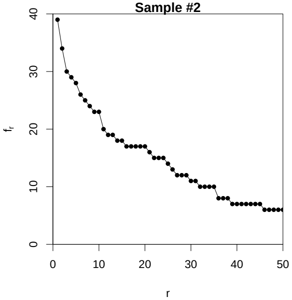
sample #2

Random variation in type-frequency lists





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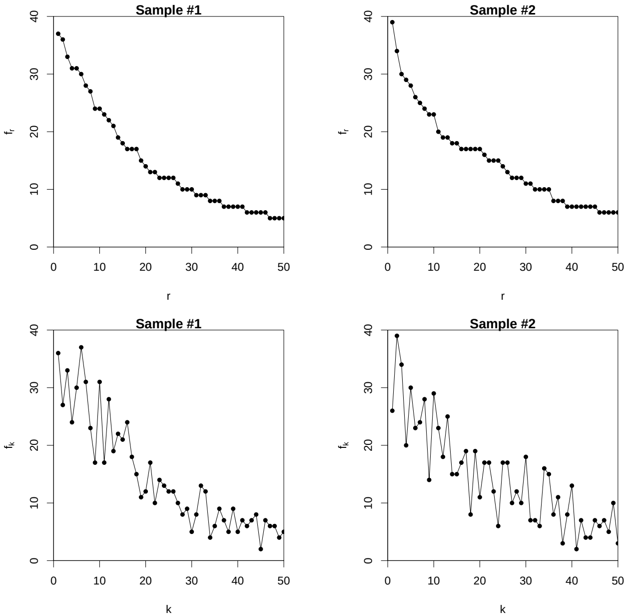

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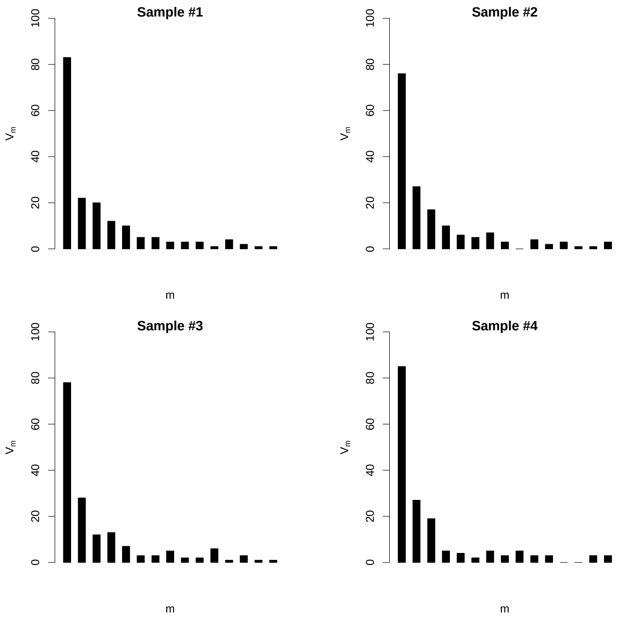
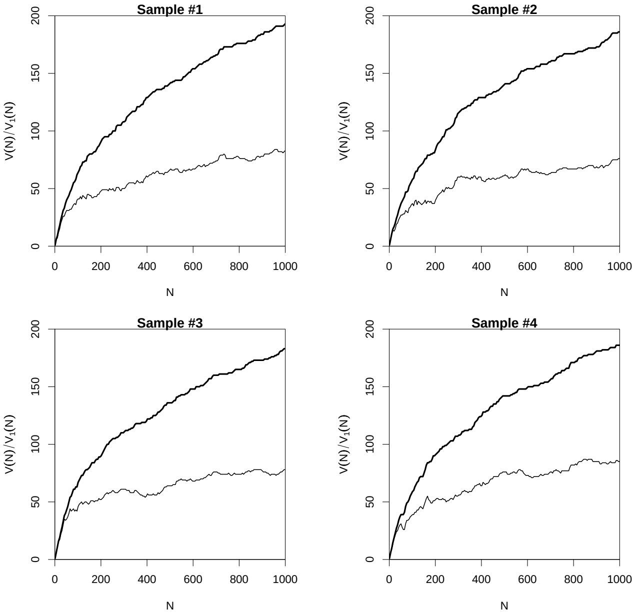
➥ considerable amount of random variation still visible

Random variation: frequency spectrum



Random variation: vocabulary growth curve



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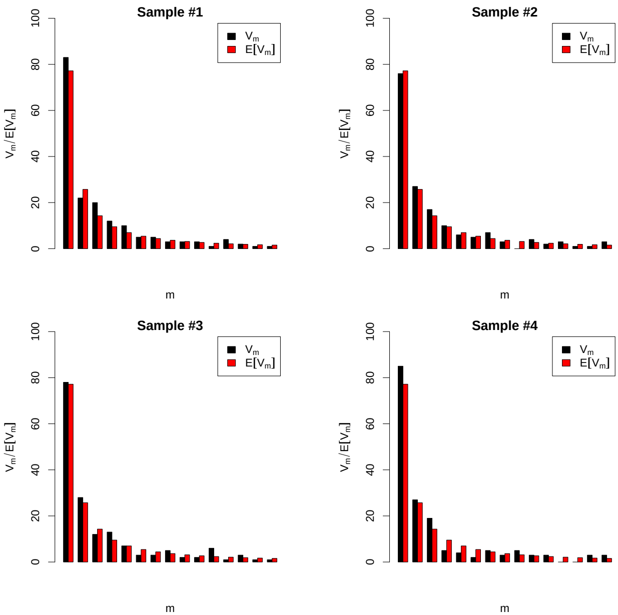
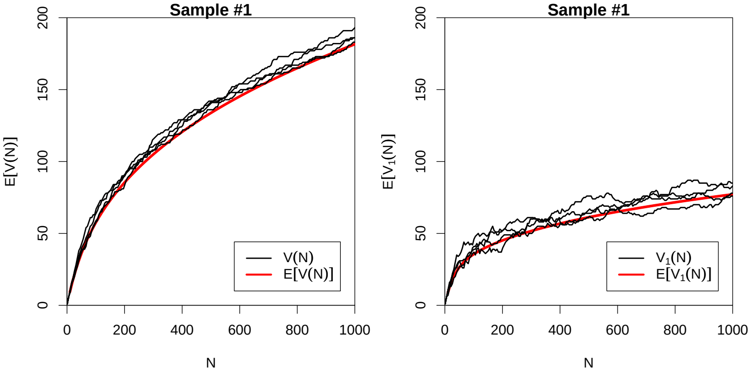
- ➥ Take the average over a large number of samples
- Such averages are called **expected values** or **expectations** in statistics (frequentist approach)
- Notation: $\mathrm{E}[V(N)]$ and $\mathrm{E}[V_m(N)]$
  - indicates that we are referring to expected values for a sample of size N
  - rather than to the specific values V and V_m observed in a particular sample or a real-world data set
- Usually we can omit the sample size: $\mathrm{E}[V]$ and $\mathrm{E}[V_m]$

The expected frequency spectrum



The expected vocabulary growth curve



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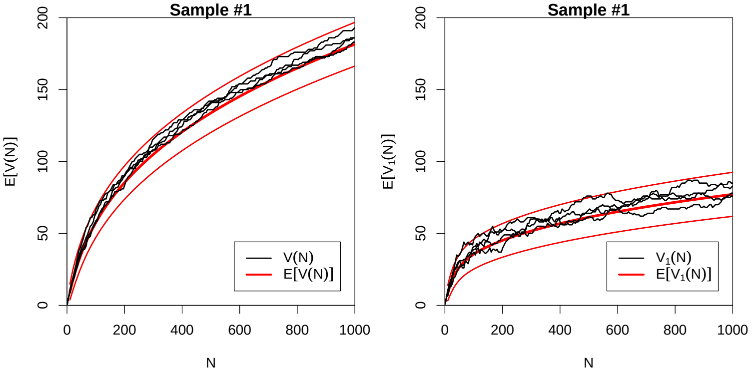
- This solution also allows us to estimate the amount of random variation ➜ **variance** and **confidence intervals**
  - example: expected VGCs with confidence intervals
  - we won't pursue variance any further in this course

Confidence intervals for the expected VGC



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- The most frequent word in English is *the* with $\pi \approx .06$
- Zipf-Mandelbrot law with $a = 1.5$ and $b = 7.5$ yields a population model where $\pi_1 \approx .06$ (by trial & error)

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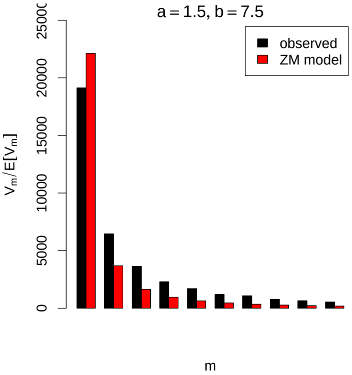
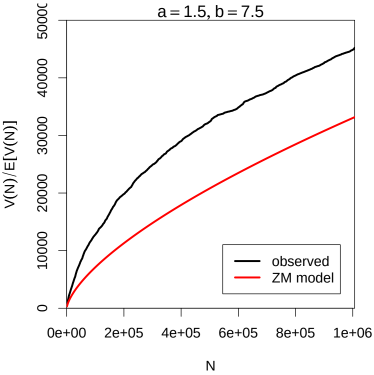
- Choose parameters so that population model matches the empirical distribution as well as possible
- E.g. by trial and error ...
  - guess parameters
  - compare model predictions for sample of size N_0 with observed data (N_0 tokens)
  - based on frequency spectrum or vocabulary growth curve
  - change parameters & repeat until satisfied
- This process is called **parameter estimation**

Parameter estimation by trial & error





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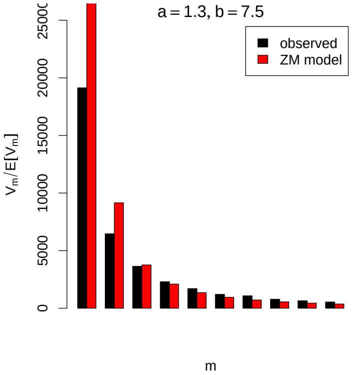

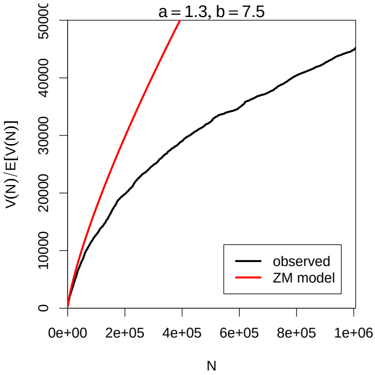

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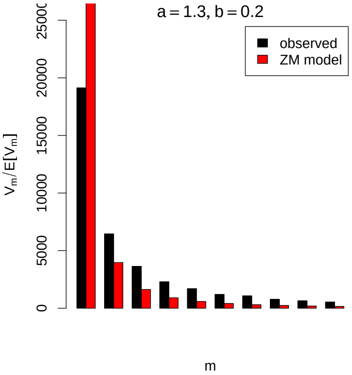

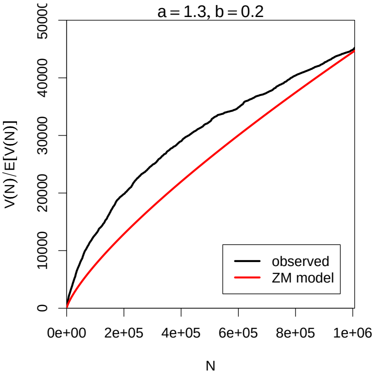

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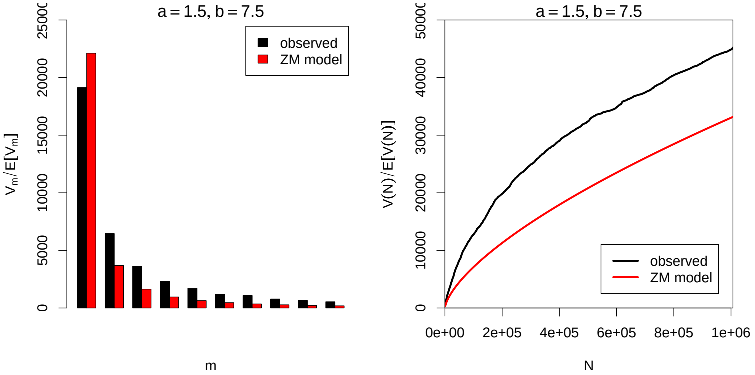

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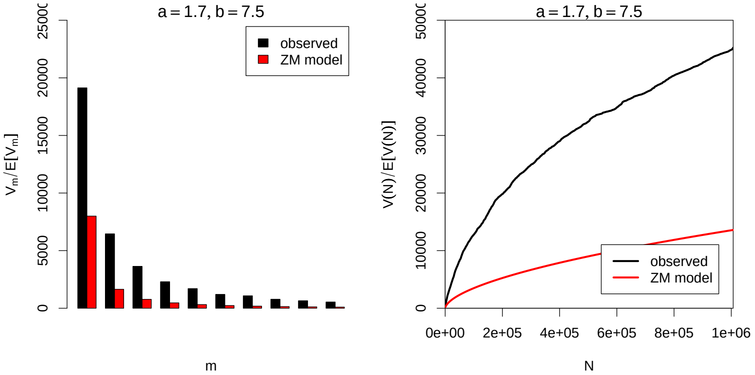

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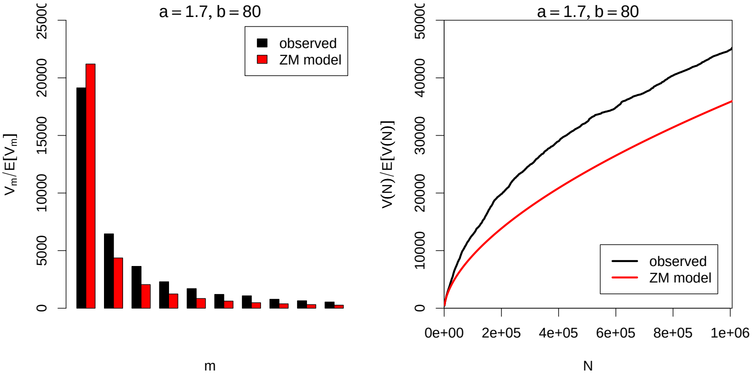

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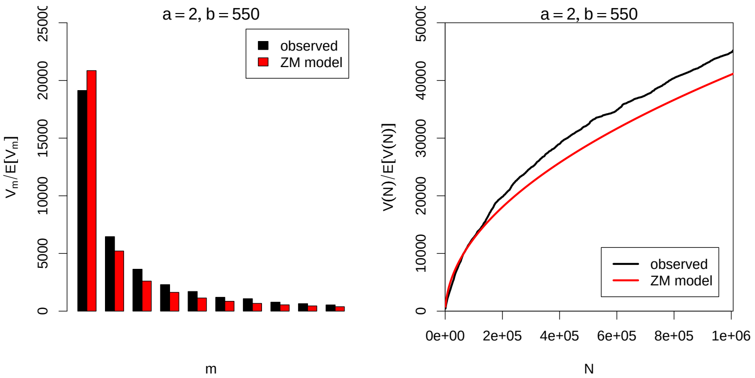

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- Need **cost function** to quantify "distance" between model expectations and observed data
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- Computer estimates parameters by automatic minimization of cost function
  - clever algorithms exist that find out quickly in which direction they have to "push" the parameters to approach the minimum
  - implemented in standard software packages

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  - absolute values of differences $\sum_{m=1}^{M} \left| V_m - \mathrm{E}[V_m] \right|$
  - mean squared error $\frac{1}{M} \sum_{m=1}^{M} \left(V_m - \mathrm{E}[V_m] \right)^2$
  - chi-squared criterion: scale by estimated variances

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- Felicitous choice of cost function and M can substantially improve the quality of the estimated model
  - It isn't a science, it's an art ...

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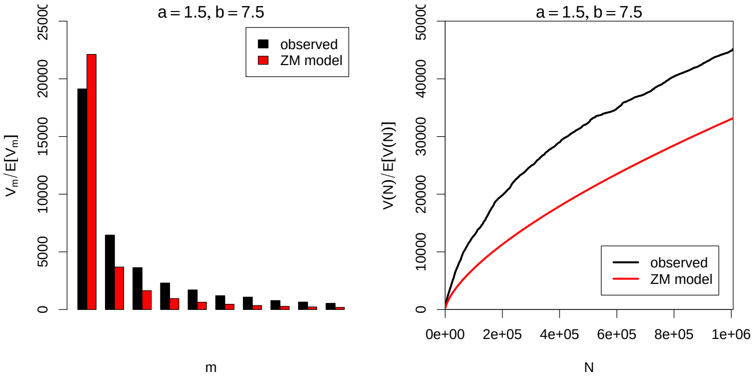
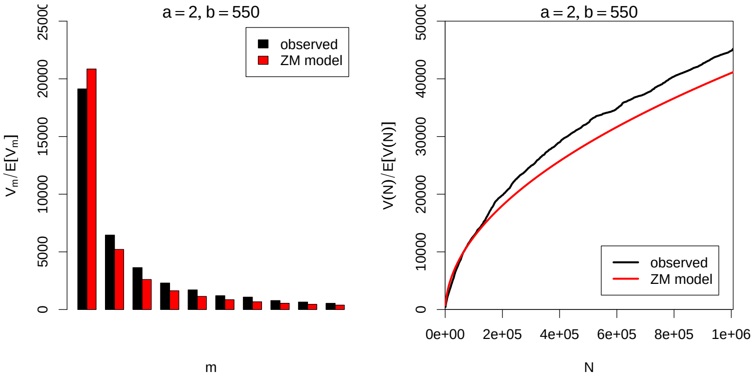
- Typically, we find $p < .001$ – but the models can still be useful for many purposes!

Mini-example (cont'd)



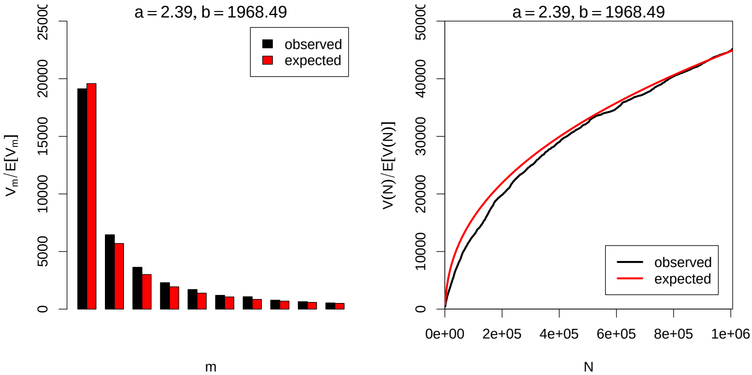
- We started with $a = 1.5$ and $b = 7.5$ (general assumptions)

Mini-example (cont'd)



- By trial & error we found $a = 2.0$ and $b = 550$

Mini-example (cont'd)



- Automatic estimation procedure: $a = 2.39$ and $b = 1968$
- Goodness-of-fit: $p \approx 0$ (but much better than before!)

Outline

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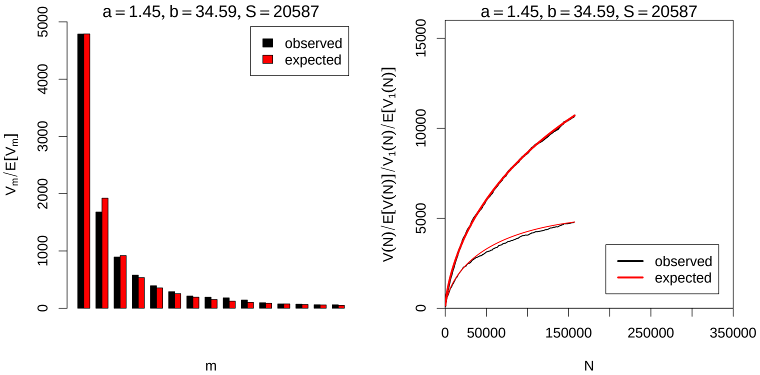
  - additional constraint: $\mathrm{E}[V(N_0)] = V(N_0)$
- Automatic parameter estimation yields $a = 1.45$, $b = 34.6$, $S = 20587$
  - population vocabulary size is extremely small
  - but this model extrapolates only the vocabulary used in *Oliver Twist*, not the full vocabulary of Charles Dickens

Results for *Oliver Twist*



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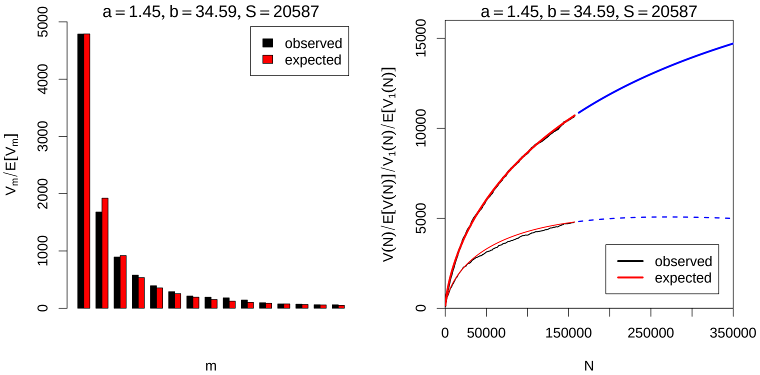

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