What Every Corpus Linguist Should Know About Type-Token Distributions and Zipf's Law

Tutorial Workshop #9, 22 July 2019

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http://zipfr.r-forge.r-project.org/lrec2018.html

CARDIFF UNIVERSITY PRIFYSGOL CAERDYDD
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Outline

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Some research questions

- How many words did Shakespeare know?
- What is the coverage of my treebank grammar on big data?
- How many typos are there on the Internet?
- Is *-ness* more productive than *-ity* in English?
- Are there differences in the productivity of nominal compounds between academic writing and novels?
- Does Dickens use a more complex vocabulary than Rowling?
- Can a decline in lexical complexity predict Alzheimer's disease?
- How frequent is a hapax legomenon from the Brown corpus?
- What is appropriate smoothing for my n-gram model?
- Who wrote the Bixby letter, Lincoln or Hay?
- How many different species of . . . are there? (Brainerd 1982)

Some research questions

- $\searrow\searrow$
- coverage estimates
- $\nearrow\nearrow$
- $\searrow\searrow$
- productivity
- lexical complexity & stylometry
- $\nearrow\nearrow$
- prior & posterior distribution
- $\nearrow\nearrow$
- $\searrow\searrow$
- unexpected applications

Type-token statistics

- These applications relate token and type counts
  - **tokens** = individual instances (occurrences)
  - **types** = distinct items
- Type-token statistics different from most statistical inference
  - not about probability of a specific event
  - but about diversity of events and their probability distribution

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- Relatively little work in statistical science
- Nor a major research topic in computational linguistics
  - very specialized, usually plays ancillary role in NLP
- Corpus linguistics: TTR & simple productivity measures
  - often applied without any statistical inference

Zipf's law (Zipf 1949)

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B) Curious relationship between rank & frequency

word	r	f	$r \cdot f$
the	1.	142,776	142,776
and	2.	100,637	201,274
be	3.	94,181	282,543
of	4.	74,054	296,216

(Dickens)

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C) Various explanations of Zipf's law

- principle of least effort (Zipf 1949)
- optimal coding system, MDL (Mandelbrot 1953, 1962)
- random sequences (Miller 1957; Li 1992; Cao *et al.* 2017)
- Markov processes ➜ n-gram models (Rouault 1978)

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D) Language evolution: birth-death-process (Simon 1955)

☞ not the main topic today!

Goals of this tutorial

- Introduce descriptive statistics, notation and terminology
- Explain mathematical foundations of LNRE models for statistical inference
- Practise application of models in R
- Discuss measures of productivity & lexical richness
- Address problems and advanced techniques

Outline

Tokens & types

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

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  type-frequency list

w	f_w
recently	1
very	5
not	3
otherwise	1
much	2
merely	2
now	1

Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

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Zipf ranking

w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1

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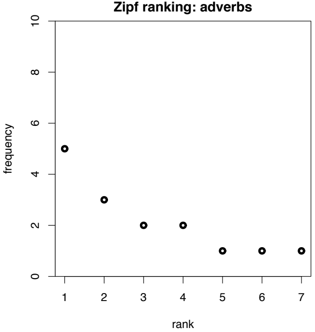

A realistic Zipf ranking: the Brown corpus

top frequencies			bottom frequencies		
r	***f***	**word**	**rank range**	***f***	**randomly selected examples**
1	69836	the	7731 – 8271	10	schedules, polynomials, bleak
2	36365	of	8272 – 8922	9	tolerance, shaved, hymn
3	28826	and	8923 – 9703	8	decreased, abolish, irresistible
4	26126	to	9704 – 10783	7	immunity, cruising, titan
5	23157	a	10784 – 11985	6	geographic, lauro, portrayed
6	21314	in	11986 – 13690	5	grigori, slashing, developer
7	10777	that	13691 – 15991	4	sheath, gaulle, ellipsoids
8	10182	is	15992 – 19627	3	mc, initials, abstracted
9	9968	was	19628 – 26085	2	thar, slackening, deluxe
10	9801	he	26086 – 45215	1	beck, encompasses, second-place

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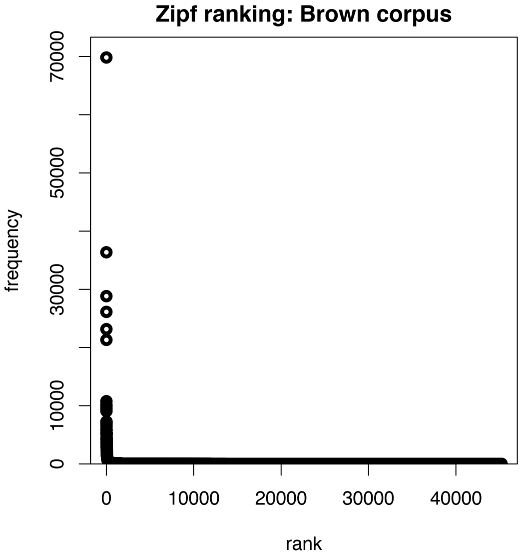

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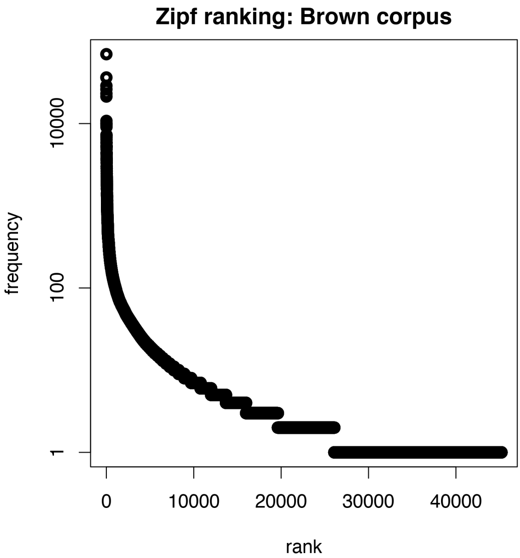

Frequency spectrum

- pool types with $f = 1$ (hapax legomena), types with $f = 2$ (dis legomena), …, $f = m$, …
- $V_1 = 3$: number of hapax legomena (*now, otherwise, recently*)
- $V_2 = 2$: number of dis legomena (*merely, much*)
- general definition: $V_m = |\{w \mid f_w = m\}|$

Zipf ranking

w	r	f_r
very	1	5
not	2	3
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much	4	2
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otherwise	6	1
recently	7	1

frequency spectrum

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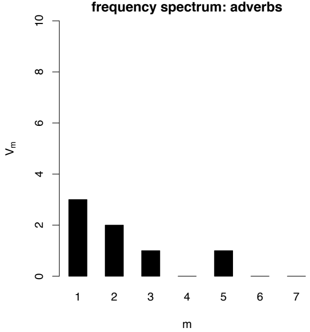

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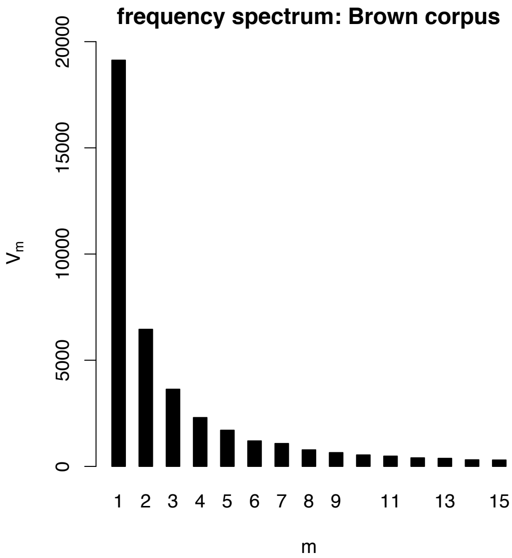

Vocabulary growth curve

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 1,\ V(N) = 1,\ V_1(N) = 1$

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- $N = 12,\ V(N) = 7,\ V_1(N) = 4$

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- $N = 15,\ V(N) = 7,\ V_1(N) = 3$

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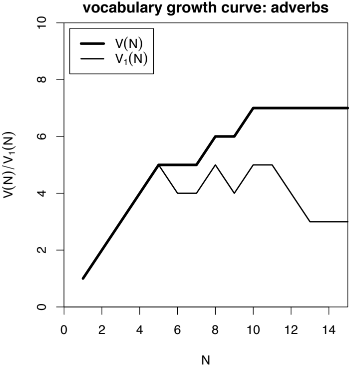
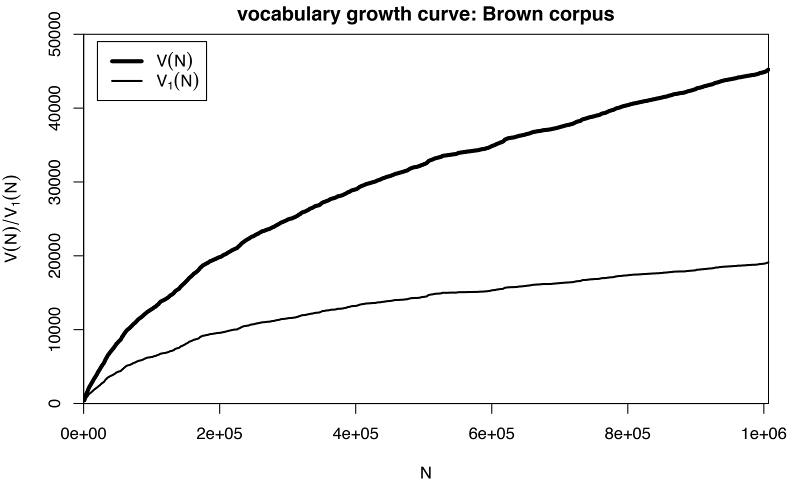
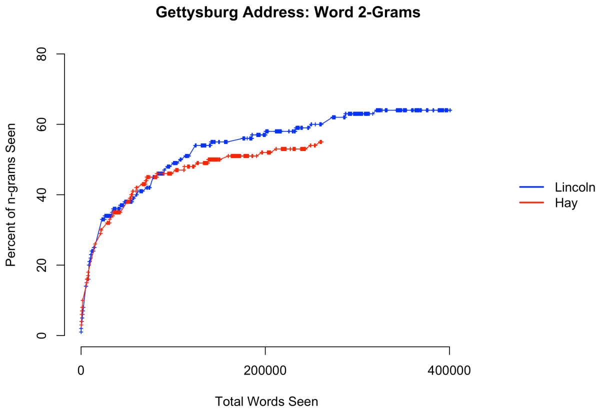


A realistic vocabulary growth curve: the Brown corpus



Vocabulary growth in authorship attribution

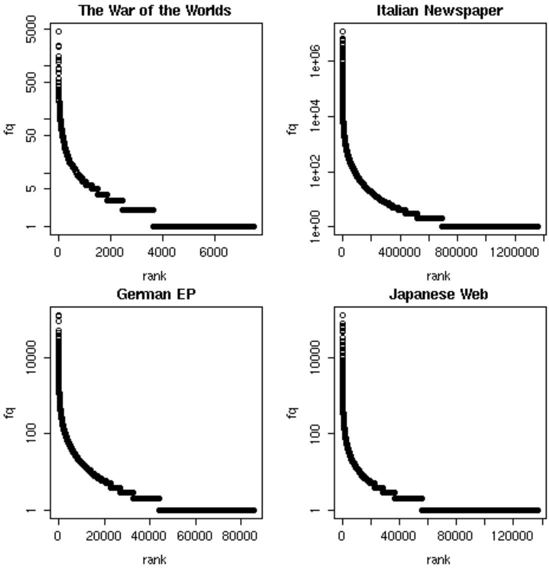
- Authorship attribution by n-gram tracing applied to the case of the Bixby letter (Grieve *et al.* 2018)
- Word or character n-grams in disputed text are compared against large "training" corpora from candidate authors



Outline

Observing Zipf's law

across languages and different linguistic units



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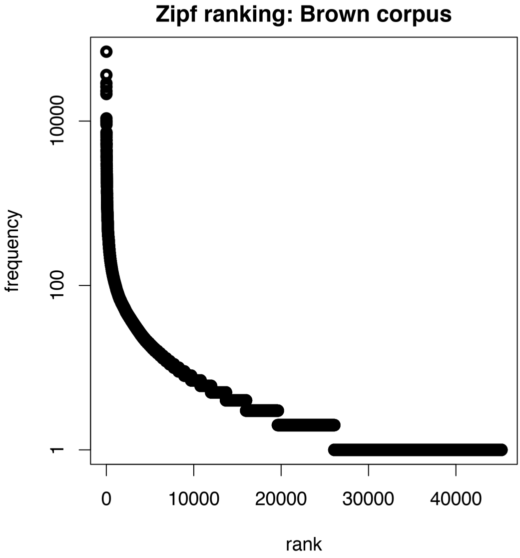

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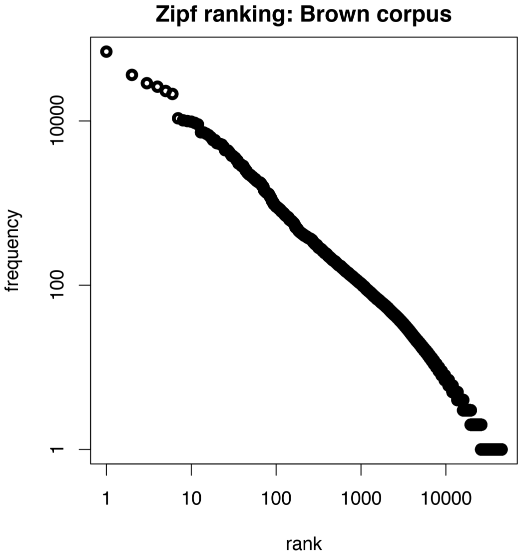

Observing Zipf's law

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- This leads to Zipf's (1949; 1965) famous law:

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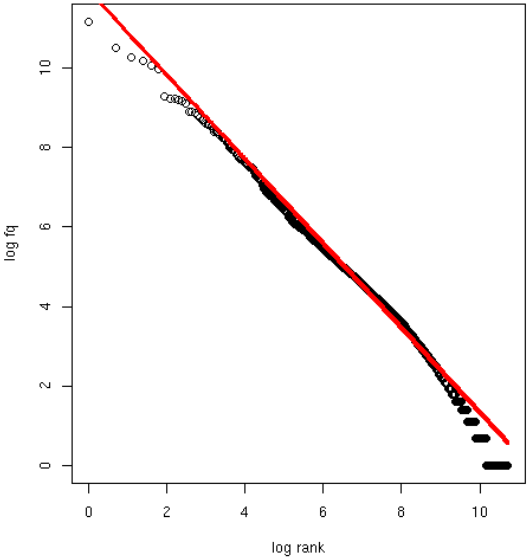
- Intuitive interpretation of a and C:
  - a is **slope** determining how fast log frequency decreases
  - $\log C$ is **intercept**, i.e. log frequency of most frequent word ($r = 1 \rightarrow \log r = 0$)

Observing Zipf's law

Least-squares fit = linear regression in log-space (Brown corpus)



Zipf-Mandelbrot law

Mandelbrot (1953, 1962)

- Mandelbrot's extra parameter:

$$f_r = \frac{C}{(r+b)^a}$$

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- Mandelbrot's extra parameter:

$$f_r = \frac{C}{(r+b)^a}$$

- Zipf's law is special case with $b = 0$
- Assuming $a = 1$, $C = 60{,}000$, $b = 1$:
  - For word with rank 1, Zipf's law predicts frequency of 60,000; Mandelbrot's variation predicts frequency of 30,000
  - For word with rank 1,000, Zipf's law predicts frequency of 60; Mandelbrot's variation predicts frequency of 59.94

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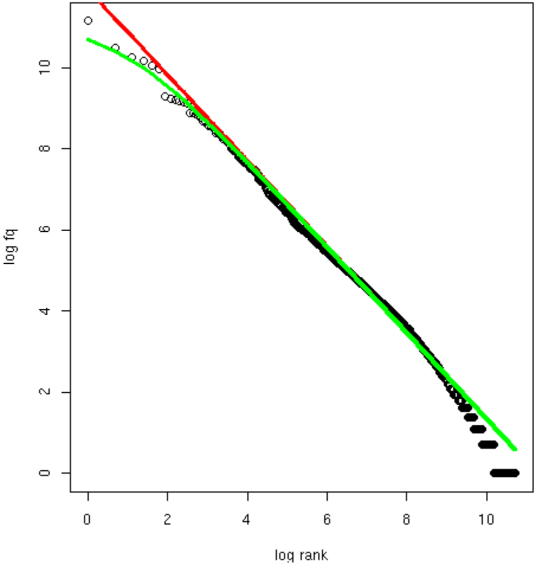
- Zipf-Mandelbrot law forms basis of statistical LNRE models
  - ZM law derived mathematically as limiting distribution of vocabulary generated by a character-level Markov process

Zipf-Mandelbrot law

Non-linear least-squares fit (Brown corpus)



Outline

zipfR

Evert and Baroni (2007)

- `http://zipfR.R-Forge.R-Project.org/`
- Conveniently available from CRAN repository
- Package vignette = gentle tutorial introduction

First steps with zipfR

- Set up a folder for this course, and make sure it is your working directory in R (preferably as an RStudio project)
- Install the most recent version of the zipfR package
  - tutorial requires version 0.7 or newer
- Package, handouts, code samples & data sets available from `http://zipfr.r-forge.r-project.org/lrec2018.html`

```
> library(zipfR)

> ?zipfR    # documentation entry point

> vignette("zipfr-tutorial")    # read the zipfR tutorial
```

Loading type-token data

- Most convenient input: sequence of tokens as text file in vertical format ("one token per line")
  - ☞ mapped to appropriate types: normalized word forms, word pairs, lemmatized, semantic class, n-gram of POS tags, ...
  - ☞ language data should always be in UTF-8 encoding!
  - ☞ large files can be compressed (`.gz`, `.bz2`, `.xz`)

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```
> adv <- readLines("brown_adverbs.txt", encoding="UTF-8")
```

```
> head(adv, 30)   # mathematically, a ''vector'' of tokens
> length(adv)     # sample size = 52,037 tokens
```

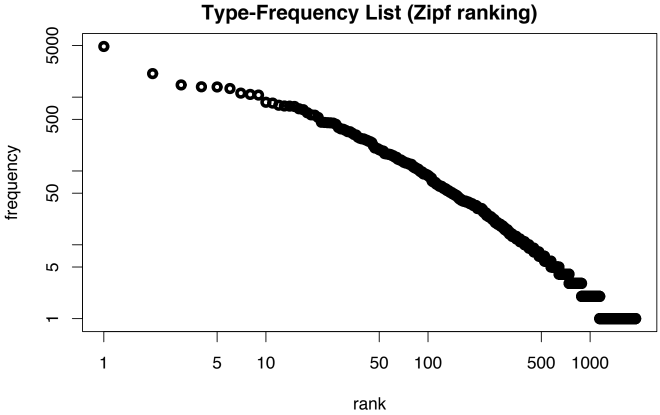
Descriptive statistics: type-frequency list

```
> adv.tfl <- vec2tfl(adv)
> adv.tfl
        k    f  type
not     1 4859   not
n't     2 2084   n't
so      3 1464    so
only    4 1381  only
then    5 1374  then
now     6 1309   now
even    7 1134  even
as      8 1089    as
        ⋮    ⋮     ⋮

     N    V
 52037 1907

> N(adv.tfl)   # sample size
> V(adv.tfl)   # type count
```

Descriptive statistics: type-frequency list

- Visualize descriptive statistics with `plot` method

```
> plot(adv.tfl)              # Zipf ranking
> plot(adv.tfl, log="xy")    # logarithmic scale recommended
```

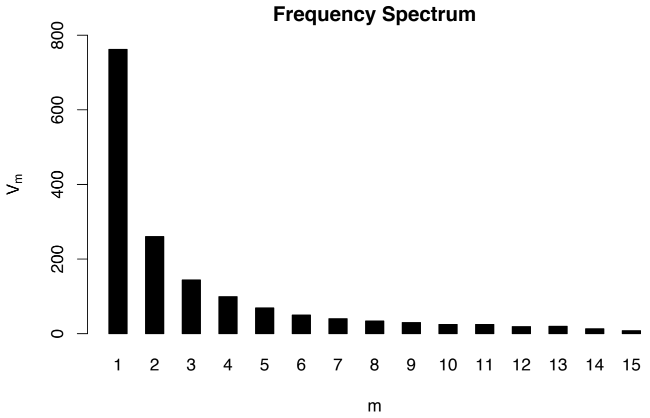


Descriptive statistics: frequency spectrum

```
> adv.spc <- tfl2spc(adv.tfl)   # or directly with vec2spc
> adv.spc
    m  Vm
1   1 762
2   2 260
3   3 144
4   4  99
5   5  69
6   6  50
7   7  40
8   8  34
    ⋮   ⋮
     N    V
 52037 1907

> N(adv.spc)   # sample size
> V(adv.spc)   # type count
```

Descriptive statistics: frequency spectrum

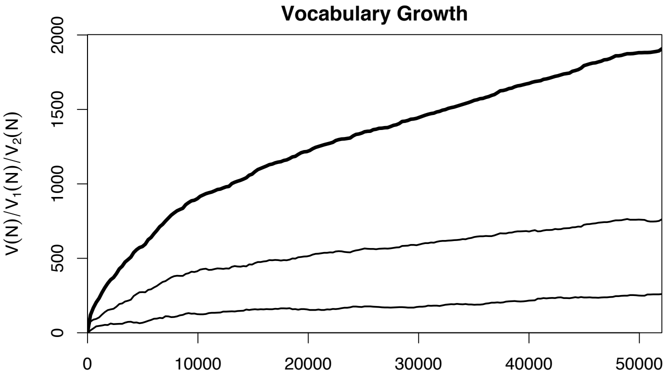
```
> plot(adv.spc)           # barplot of frequency spectrum
> ?plot.spc               # see help page for further options
```



Descriptive statistics: vocabulary growth

- VGC lists vocabulary size $V(N)$ at different sample sizes N
- Optionally also spectrum elements $V_m(N)$ up to `m.max`

```
> adv.vgc <- vec2vgc(adv, m.max=2)
> plot(adv.vgc, add.m=1:2) # plot all three VGCs
```



Further example data sets

`?Brown`	words from Brown corpus
`?BrownSubsets`	various subsets
`?Dickens`	words from novels by Charles Dickens
`?ItaPref`	Italian word-formation prefixes
`?TigerNP`	NP and PP patterns from German Tiger treebank
`?Baayen2001`	frequency spectra from Baayen (2001)
`?EvertLuedeling2001`	German word-formation affixes (manually corrected data from Evert and Lüdeling 2001)

Practice:

- Explore these data sets with descriptive statistics
- Try different plot options (from help pages `?plot.tfl`, `?plot.spc`, `?plot.vgc`)

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  - ☞ significance of differences in productivity

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  - ☞ statistical inference from sample to population
  - ☞ significance of differences in productivity
- Discrete frequency counts are difficult to capture with generalizations such as Zipf's law
  - ☞ Zipf's law predicts many impossible types with $1 < f_r < 2$
  - ☞ population does not suffer from such quantization effects

LNRE models

- This tutorial introduces the state-of-the-art LNRE approach proposed by Baayen (2001)
  - LNRE = Large Number of Rare Events
- LNRE uses various approximations and simplifications to obtain a tractable and elegant model
- Of course, we could also estimate the precise discrete distributions using MCMC simulations, but ...
  1. LNRE model usually minor component of complex procedure
  2. often applied to very large samples ($N > 1$ M tokens)
  3. still better than naive least-squares regression on Zipf ranking

The LNRE population

- Population: set of S types w_i with occurrence **probabilities** π_i
- $S =$ **population diversity** can be finite or infinite ($S = \infty$)
- Not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$

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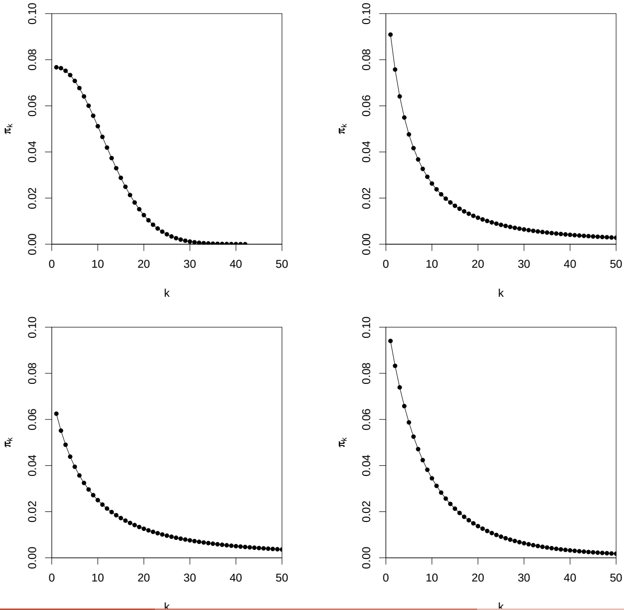
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$

- Need **parametric** statistical **model** to describe full population (esp. for $S = \infty$), i.e. a function $i \mapsto \pi_i$
  - type probabilities π_i cannot be estimated reliably from a sample, but parameters of this function can
  - NB: population index $i \neq$ Zipf rank r

What should the population look like?



Zipf-Mandelbrot law as a population model

- Zipf-Mandelbrot law for type probabilities:

$$\pi_i := \frac{C}{(i + b)^a}$$

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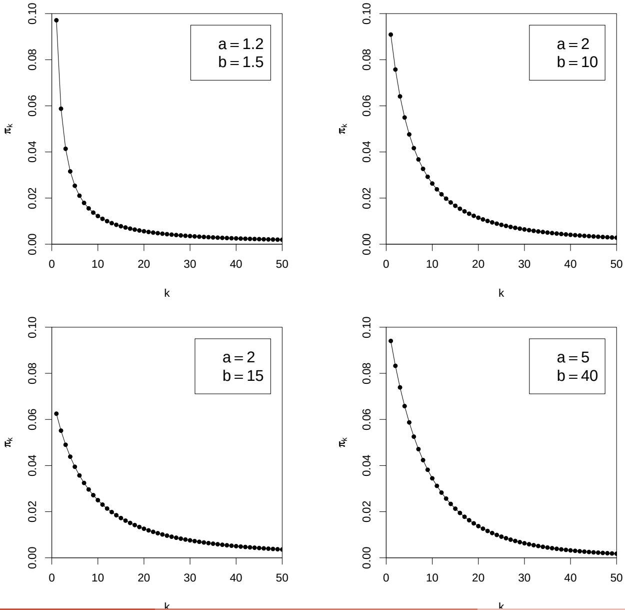
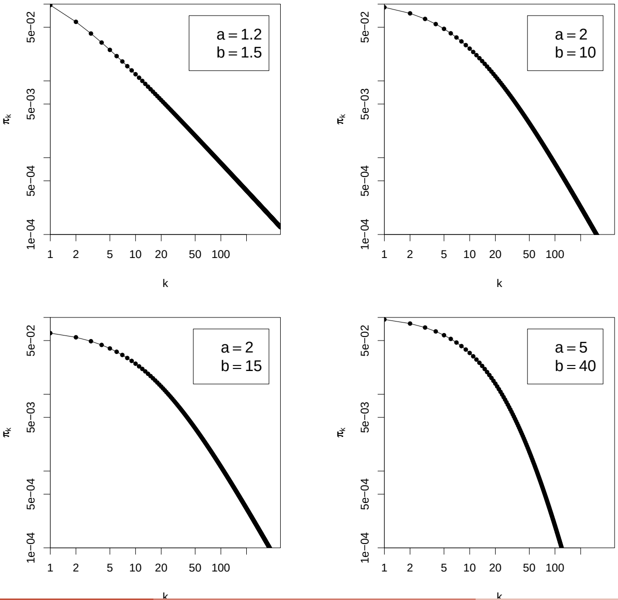
- This is the **Zipf-Mandelbrot** population model (Evert 2004)
  - **ZM** for Zipf-Mandelbrot model ($S = \infty$)
  - **fZM** for finite Zipf-Mandelbrot model

The parameters of the Zipf-Mandelbrot model

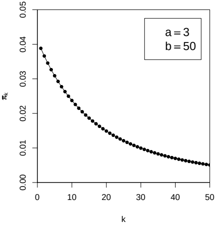


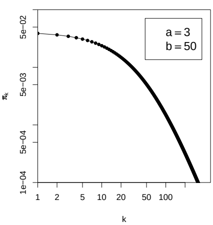
The parameters of the Zipf-Mandelbrot model



Sampling from a population model

Assume we believe that the population we are interested in can be described by a Zipf-Mandelbrot model:





Use computer simulation to generate random samples:

- Draw N tokens from the population such that in each step, type w_i has probability π_i to be picked
- This allows us to make predictions for samples (= corpora) of arbitrary size N

Sampling from a population model

#1: 1 42 34 23 108 18 48 18 1 ...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...
#3:	2	11	105	21	11	17	17	1	16	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...
#3:	2	11	105	21	11	17	17	1	16	...
#4:	44	3	110	34	223	2	25	20	28	...
#5:	24	81	54	11	8	61	1	31	35	...
#6:	3	65	9	165	5	42	16	20	7	...
#7:	10	21	11	60	164	54	18	16	203	...
#8:	11	7	147	5	24	19	15	85	37	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Samples: type frequency list & spectrum

rank r	f_r	type i
1	37	6
2	36	1
3	33	3
4	31	7
5	31	10
6	30	5
7	28	12
8	27	2
9	24	4
10	24	16
11	23	8
12	22	14
⋮	⋮	⋮

m	V_m
1	83
2	22
3	20
4	12
5	10
6	5
7	5
8	3
9	3
10	3
⋮	⋮

sample #1

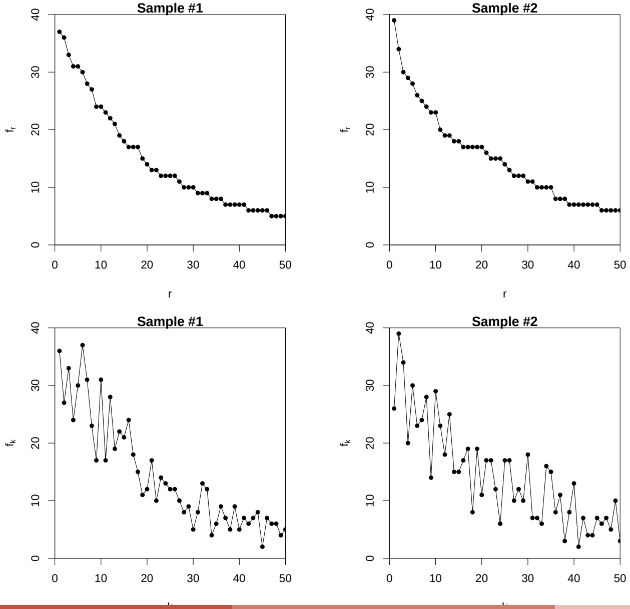
Samples: type frequency list & spectrum

rank r	f_r	type i
1	39	2
2	34	3
3	30	5
4	29	10
5	28	8
6	26	1
7	25	13
8	24	7
9	23	6
10	23	11
11	20	4
12	19	17
$\vdots$	$\vdots$	$\vdots$

m	V_m
1	76
2	27
3	17
4	10
5	6
6	5
7	7
8	3
10	4
11	2
$\vdots$	$\vdots$

sample #2

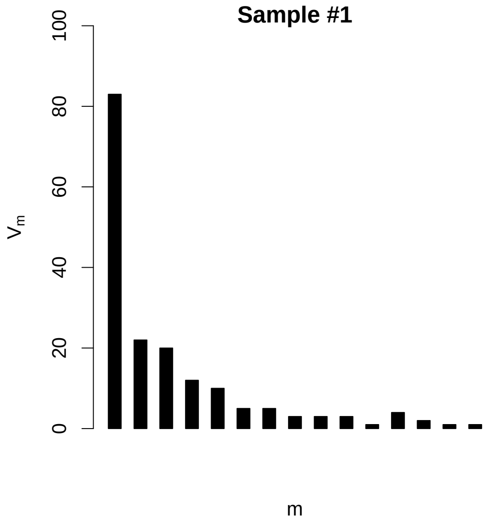
Random variation in type-frequency lists



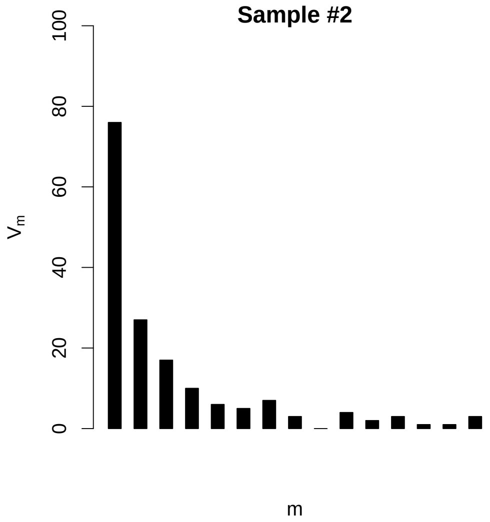
$r \leftrightarrow f_r$

$i \leftrightarrow f_i$

Random variation: frequency spectrum



Random variation: frequency spectrum



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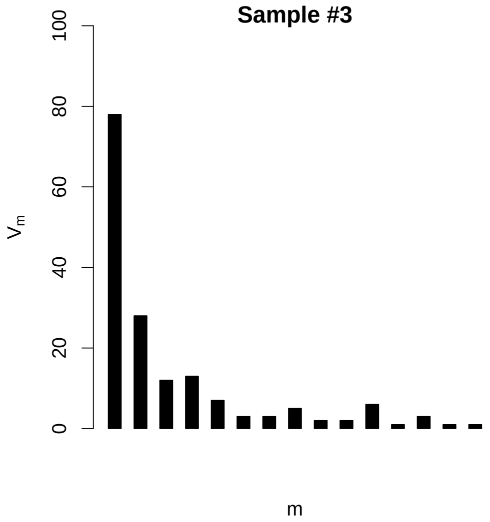
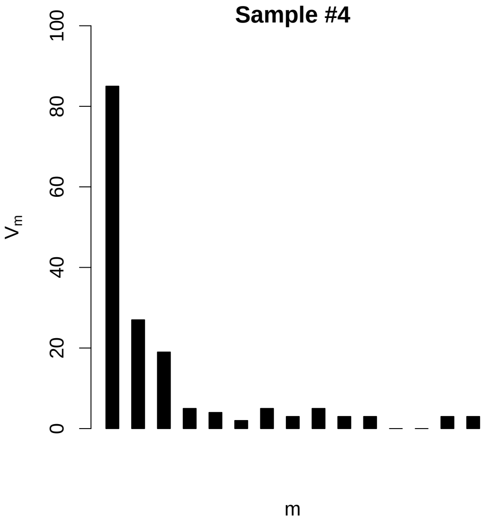
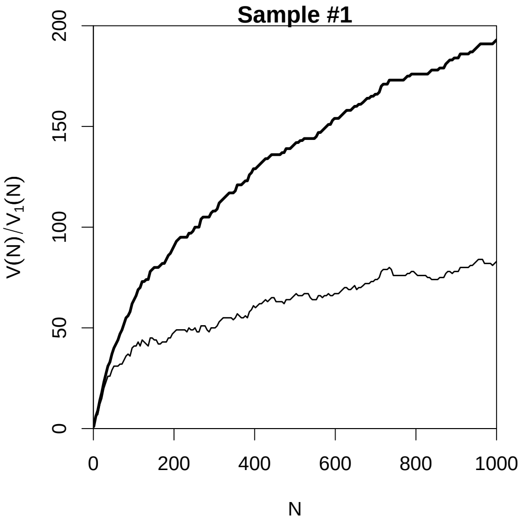


Random variation: frequency spectrum



Random variation: vocabulary growth curve



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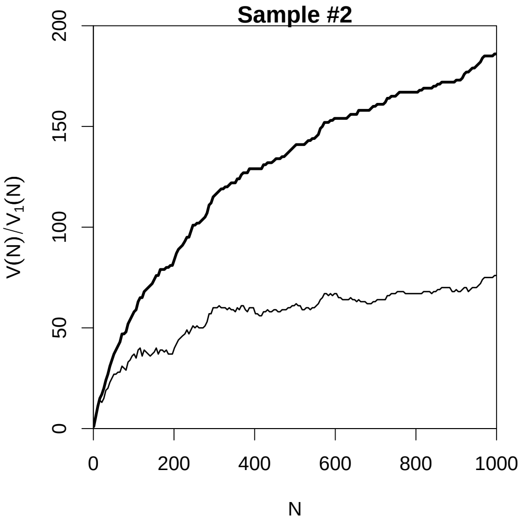

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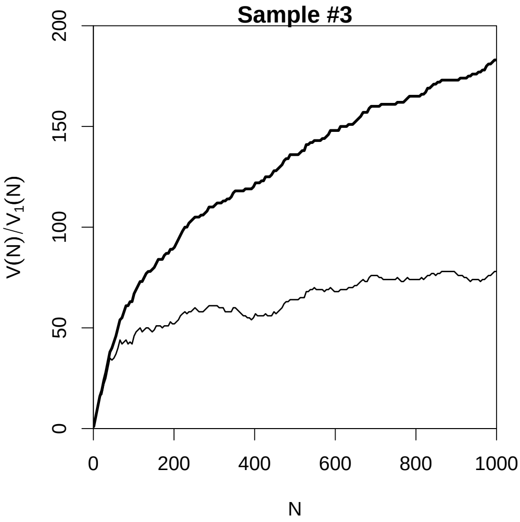

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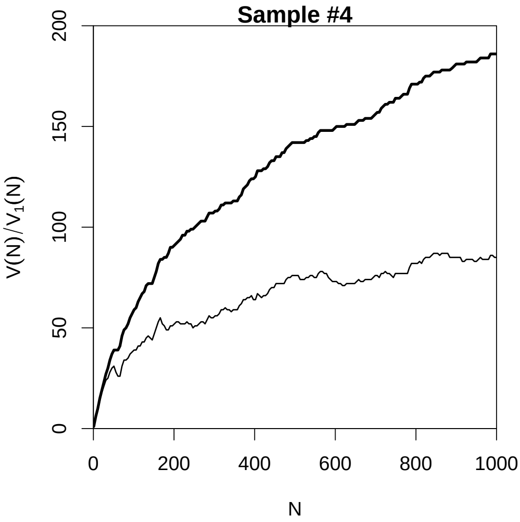


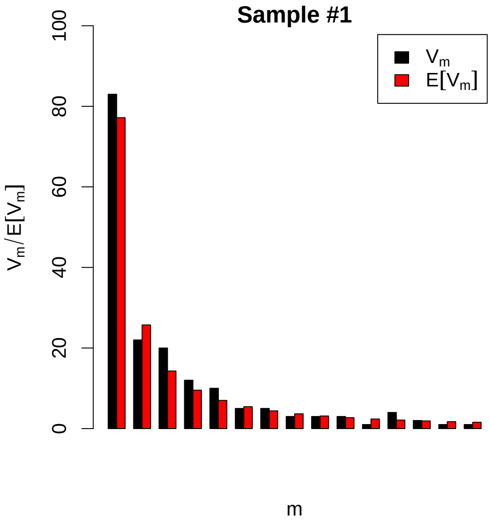
Expected values

- There is no reason why we should choose a particular sample to compare to the real data or make a prediction – each one is equally likely or unlikely
- Take the average over a large number of samples, called **expected value** or **expectation** in statistics

- Notation: $\mathrm{E}\big[V(N)\big]$ and $\mathrm{E}\big[V_m(N)\big]$
  - indicates that we are referring to expected values for a sample of size N
  - rather than to the specific values V and V_m observed in a particular sample or a real-world data set

- Expected values can be calculated efficiently *without* generating thousands of random samples

The expected frequency spectrum



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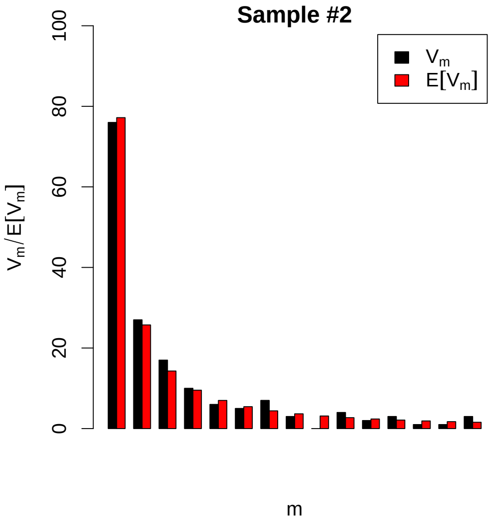

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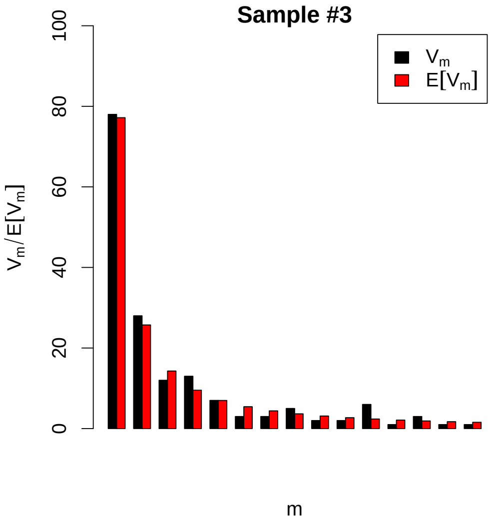

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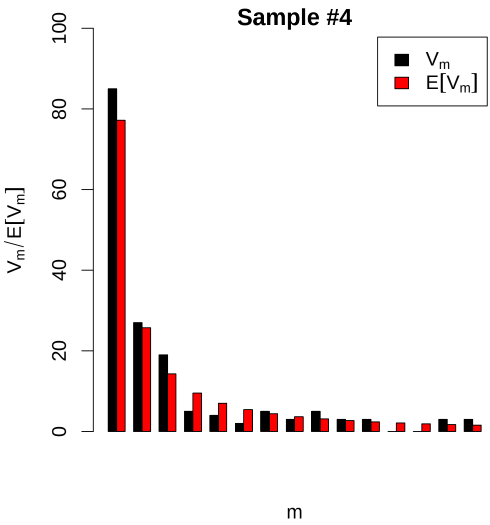

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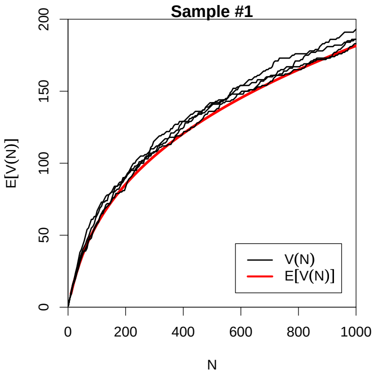
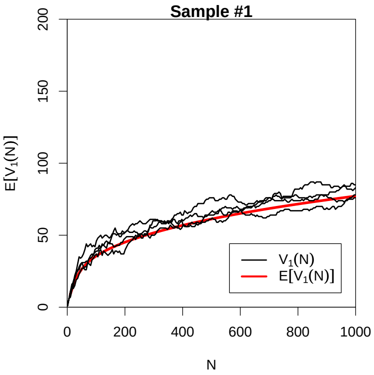
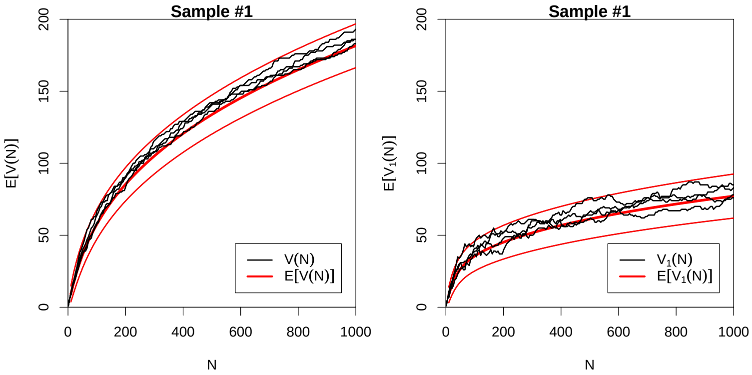




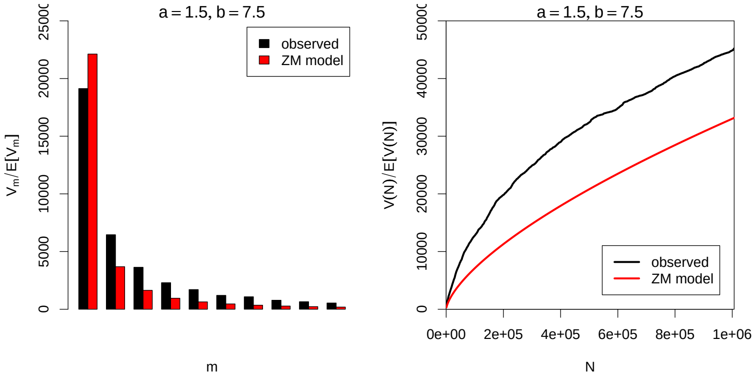
Prediction intervals for the expected VGC



"Confidence intervals" indicate predicted sampling distribution:

☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines

Parameter estimation by trial & error



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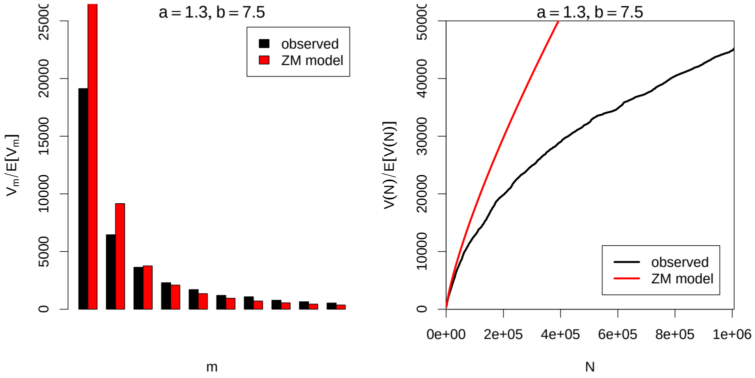

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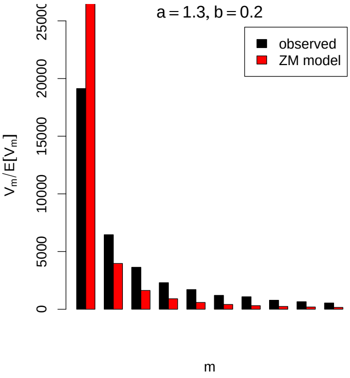

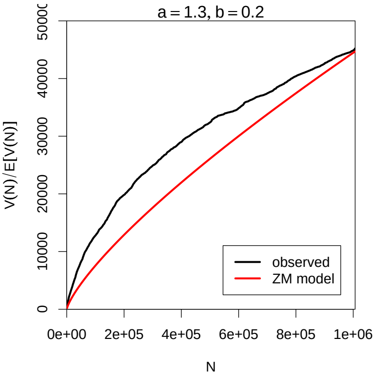

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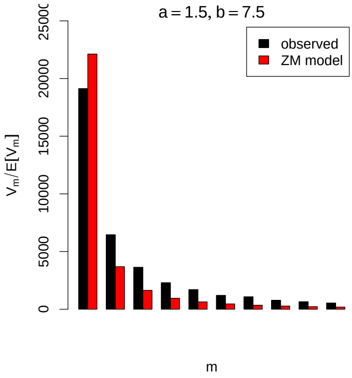

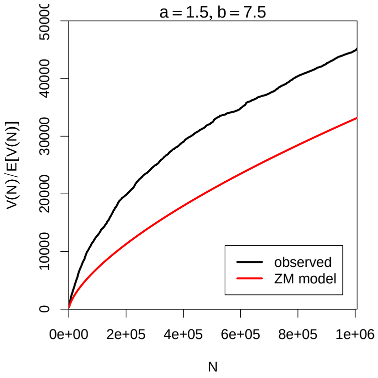

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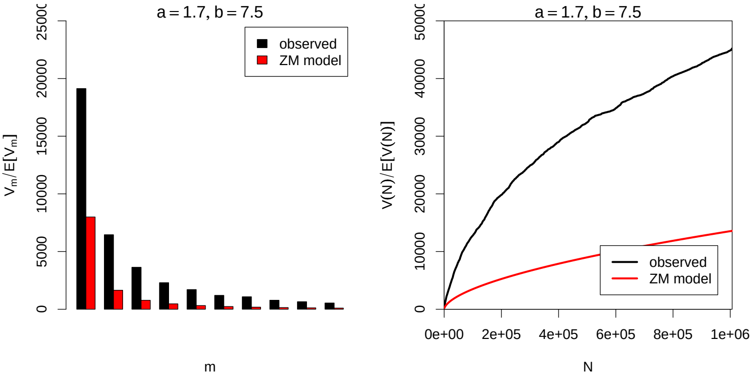

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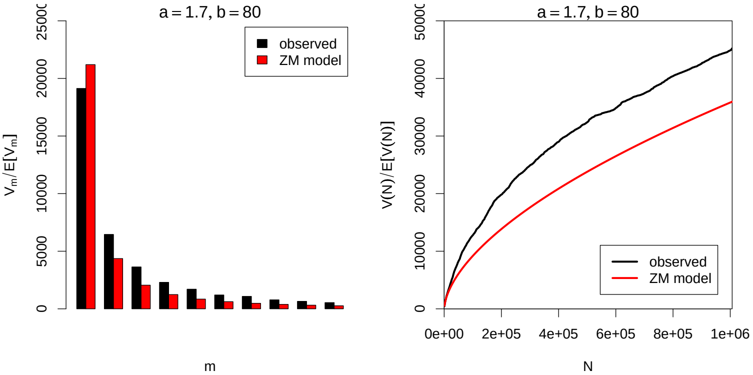

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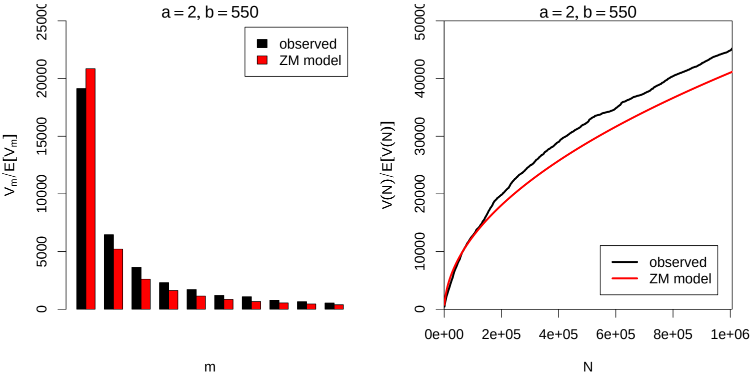
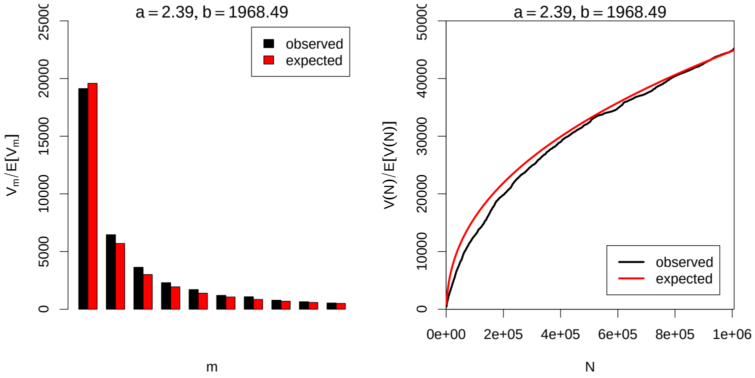


Automatic parameter estimation



- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure: $a = 2.39$ and $b = 1968$

Outline

The sampling model

- Draw random sample of N tokens from LNRE population
- Sufficient statistic: set of type frequencies $\{f_i\}$
  - because tokens of random sample have no ordering
- Joint **multinomial** distribution of $\{f_i\}$:

$$\Pr(\{f_i = k_i\} \mid N) = \frac{N!}{k_1! \cdots k_S!} \pi_1^{k_1} \cdots \pi_S^{k_S}$$

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- **Approximation:** do not condition on fixed sample size N
  - N is now the average (expected) sample size
- Random variables f_i have **independent Poisson** distributions:

$$\Pr(f_i = k_i) = e^{-N\pi_i} \frac{(N\pi_i)^{k_i}}{k_i!}$$

Frequency spectrum

- Key problem: we cannot determine f_i in observed sample
  - because we don't know which type w_i is
  - recall that population ranking $f_i \neq$ Zipf ranking f_r
- Use spectrum $\{V_m\}$ and sample size V as statistics
  - contains all information we have about observed sample

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$$V_m = \sum_{i=1}^{S} I_{[f_i=m]}$$

$$V = \sum_{i=1}^{S} I_{[f_i>0]} = \sum_{i=1}^{S} \bigl(1 - I_{[f_i=0]}\bigr)$$

The expected spectrum

- It is easy to compute expected values for the frequency spectrum (and variances because the f_i are independent)

$$\mathrm{E}[I_{[f_i=m]}] = \Pr(f_i = m) = e^{-N\pi_i} \frac{(N\pi_i)^m}{m!}$$

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$$
\begin{aligned}
\mathrm{E}[I_{[f_i=m]}] &= \Pr(f_i = m) = e^{-N\pi_i}\frac{(N\pi_i)^m}{m!} \\
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\end{aligned}
$$

- NB: V_m and V are not independent because they are derived from the same random variables f_i

Sampling distribution of V_m and V

- Joint sampling distribution of $\{V_m\}$ and V is complicated
- **Approximation:** V and $\{V_m\}$ asymptotically follow a **multivariate normal** distribution
  - motivated by the multivariate central limit theorem: sum of many independent variables $I_{[f_i=m]}$
- Usually limited to first spectrum elements, e.g. $V_1, \ldots, V_{15}$
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  - approximation of discrete V_m by continuous distribution suitable only if $\mathrm{E}[V_m]$ is sufficiently large
- Parameters of multivariate normal: $\boldsymbol{\mu} = (\mathrm{E}[V], \mathrm{E}[V_1], \mathrm{E}[V_2], \ldots)$ and $\boldsymbol{\Sigma}$ = covariance matrix

$$\Pr\big((V, V_1, \ldots, V_k) = \mathbf{v}\big) \sim \frac{e^{-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{v}-\boldsymbol{\mu})}}{\sqrt{(2\pi)^{k+1} \det \boldsymbol{\Sigma}}}$$

Type density function

- Discrete sums of probabilities in $\mathrm{E}[V]$, $\mathrm{E}[V_m]$, ... are inconvenient and computationally expensive
- **Approximation:** continuous **type density function** $g(\pi)$

$$
\left|\{w_i \mid a \leq \pi_i \leq b\}\right| = \int_a^b g(\pi)\, d\pi
$$

$$
\sum\{\pi_i \mid a \leq \pi_i \leq b\} = \int_a^b \pi g(\pi)\, d\pi
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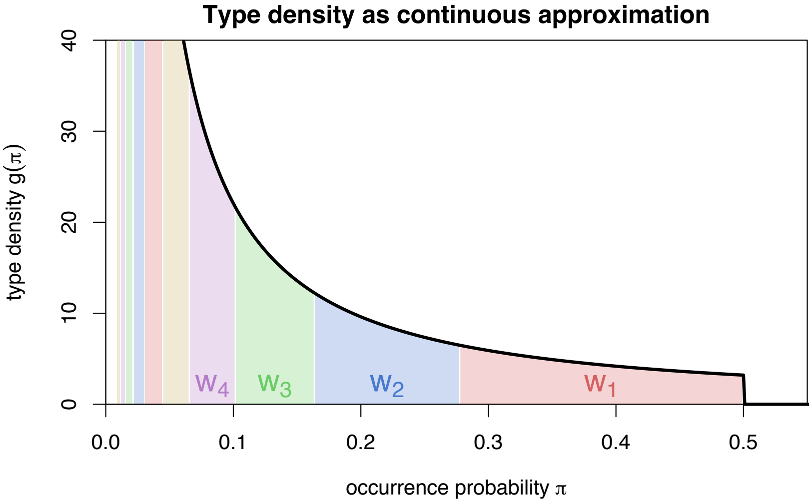
- Normalization constraint:

$$\int_0^\infty \pi g(\pi)\, d\pi = 1$$

- Good approximation for low-probability types, but probability mass of $w_1, w_2, \ldots$ "smeared out" over range

Type density function



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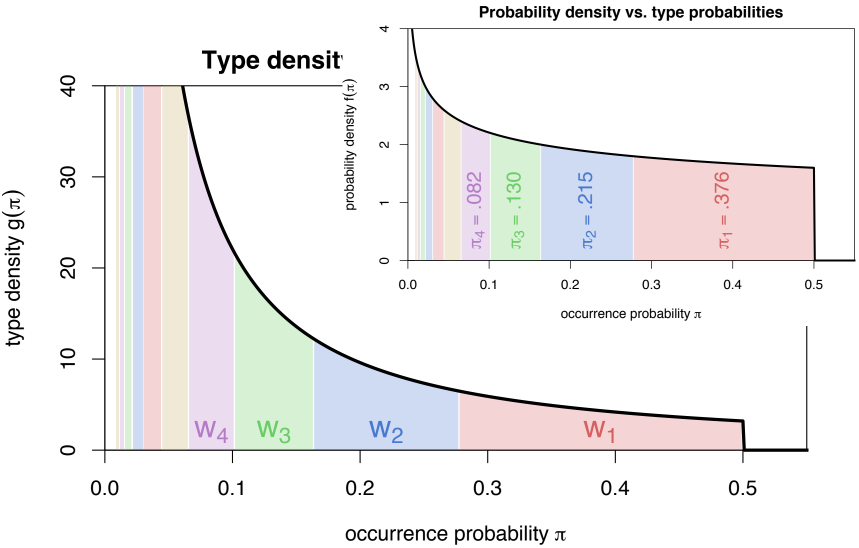

ZM and fZM as LNRE models

- Discrete Zipf-Mandelbrot population

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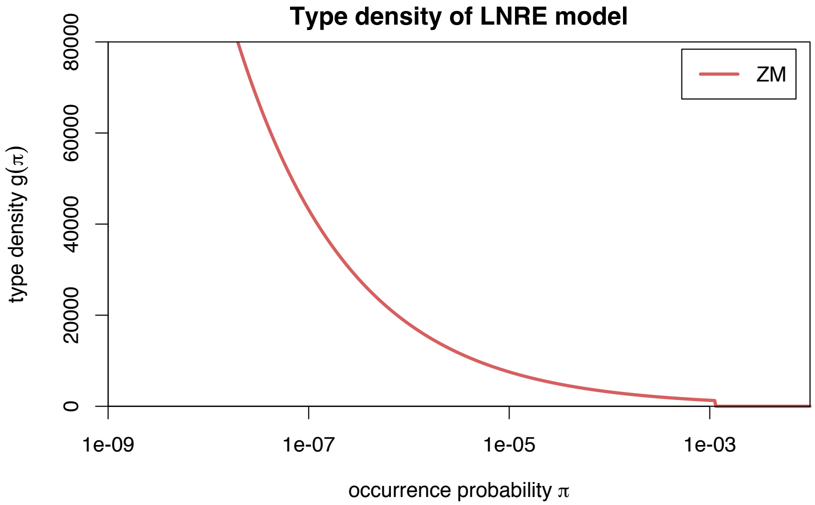
with parameters
  - $\alpha = 1/a$ $(0 < \alpha < 1)$
  - $B = (1-\alpha)/(b \cdot \alpha)$
  - $0 \leq A < B$ determines S (ZM with $S = \infty$ for $A = 0$)

☞ C is a normalization factor, not a parameter

ZM and fZM as LNRE models



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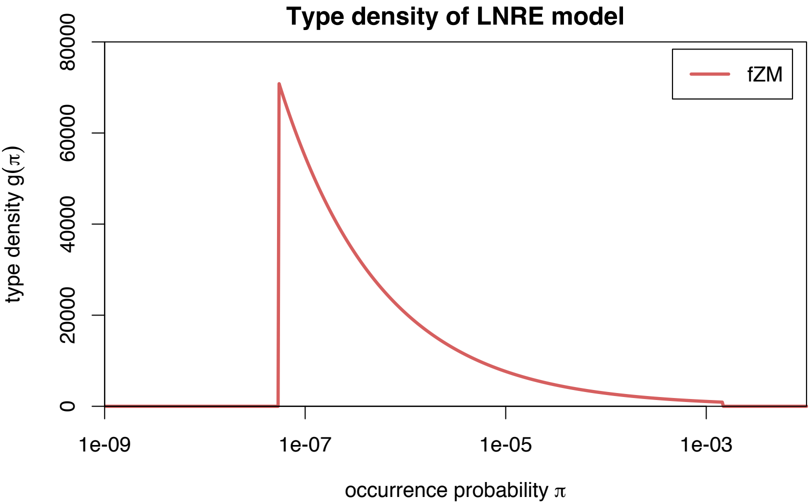

Expectations as integrals

- Expected values can now be expressed as integrals over $g(\pi)$

$$\mathrm{E}[V_m] = \int_0^\infty \frac{(N\pi)^m}{m!} e^{-N\pi} g(\pi)\, d\pi$$
$$\mathrm{E}[V] = \int_0^\infty (1 - e^{-N\pi}) g(\pi)\, d\pi$$

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- Reduce to simple closed form for ZM with $b = 0$ ($\rightarrow$ $B = \infty$)

$$\mathrm{E}[V_m] = \frac{C}{m!} \cdot N^\alpha \cdot \Gamma(m - \alpha)$$
$$\mathrm{E}[V] = C \cdot N^\alpha \cdot \frac{\Gamma(1 - \alpha)}{\alpha}$$

- fZM and general ZM with incomplete Gamma function

Parameter estimation from training corpus

- For ZM, $\alpha = \frac{\mathrm{E}[V_1]}{\mathrm{E}[V]} \approx \frac{V_1}{V}$ can be estimated directly, but prone to overfitting
- General parameter fitting by **MLE**: maximize likelihood of observed spectrum **v**

$$\max_{\alpha,A,B} \Pr\bigl((V, V_1, \ldots, V_k) = \mathbf{v} \bigm| \alpha, A, B\bigr)$$

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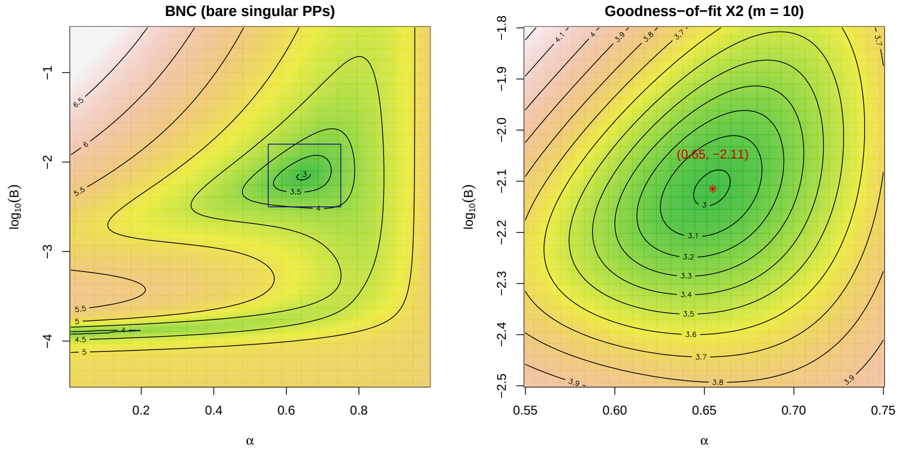
$$\max_{\alpha,A,B} \Pr\left((V, V_1, \ldots, V_k) = \mathbf{v} \,\middle|\, \alpha, A, B\right)$$

- Multivariate normal approximation:

$$\min_{\alpha,A,B} (\mathbf{v} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{v} - \boldsymbol{\mu})$$

- Minimization by gradient descent (BFGS, CG) or simplex search (Nelder-Mead)

Parameter estimation from training corpus



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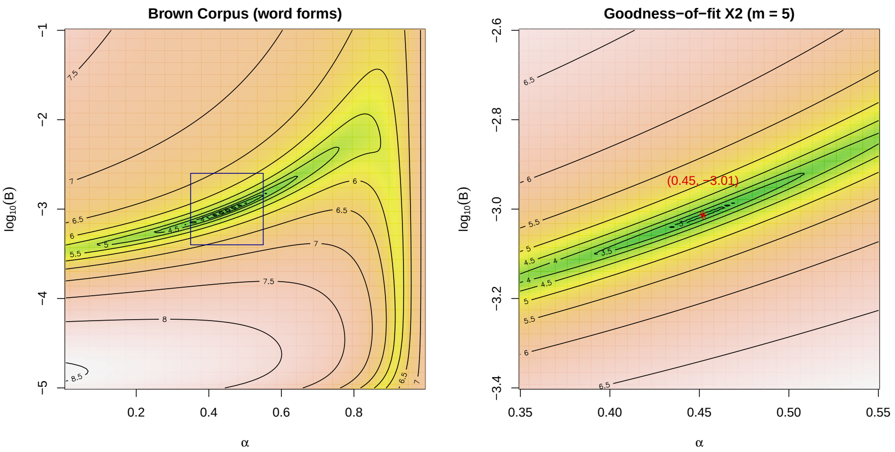

Goodness-of-fit
(Baayen 2001, Sec. 3.3)

- How well does the fitted model explain the observed data?
- For multivariate normal distribution:

$$X^2 = (\mathbf{V} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{V} - \boldsymbol{\mu}) \sim \chi^2_{k+1}$$

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➥ Multivariate chi-squared test of **goodness-of-fit**
  - replace $\mathbf{V}$ by observed $\mathbf{v}$ ➜ test statistic x^2
  - must reduce df $= k + 1$ by number of estimated parameters

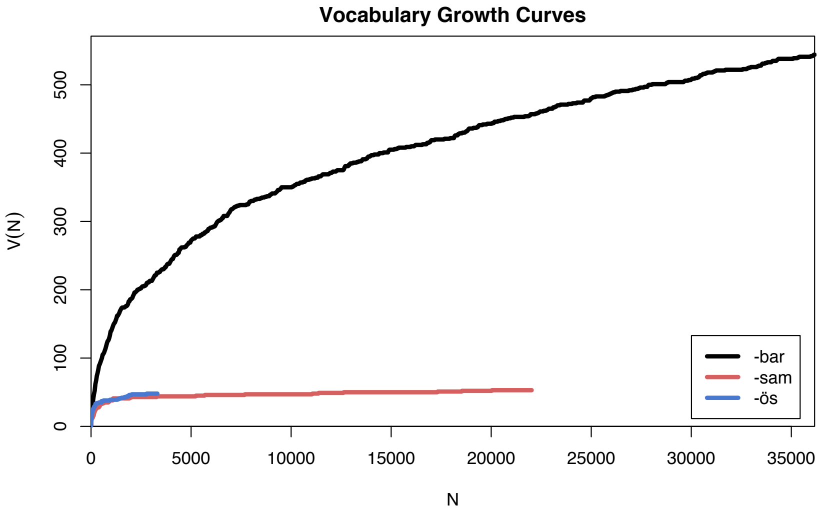
- NB: significant rejection of the LNRE model for $p < .05$

Coffee break!

Outline

Measuring morphological productivity

example from Evert and Lüdeling (2001)



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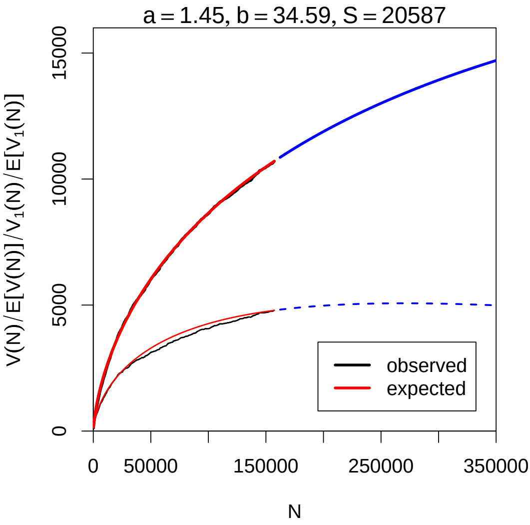

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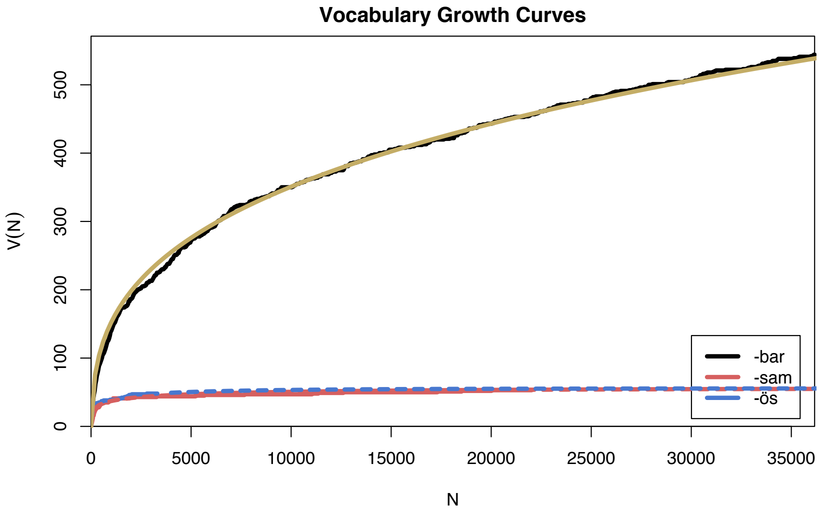

Quantitative measures of productivity

(Tweedie and Baayen 1998; Baayen 2001)

- Baayen's (1991) productivity index $\mathcal{P}$
  (slope of vocabulary growth curve)

$$\mathcal{P} = \frac{V_1}{N}$$

- TTR = type-token ratio

$$\mathrm{TTR} = \frac{V}{N}$$

- Zipf-Mandelbrot slope

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$$C = \frac{\log V}{\log N}$$

- Yule (1944) / Simpson (1949)

$$K = 10\,000 \cdot \frac{\sum_m m^2 V_m - N}{N^2}$$

- Guiraud (1954)

$$R = \frac{V}{\sqrt{N}}$$

- Sichel (1975)

$$S = \frac{V_2}{V}$$

- Honoré (1979)

$$H = \frac{\log N}{1 - \frac{V_1}{V}}$$

Productivity measures for bare singulars in the BNC

	spoken	written
V	2,039	12,876
N	6,766	85,750
K	86.84	28.57
R	24.79	43.97
S	0.13	0.15
C	0.86	0.83
$\mathcal{P}$	0.21	0.08
TTR	0.301	0.150
a	1.18	1.27
pop. S	15,958	36,874

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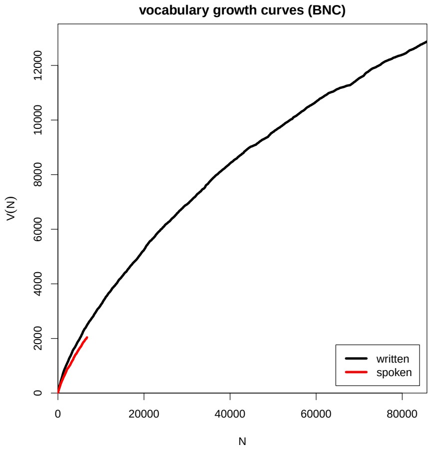
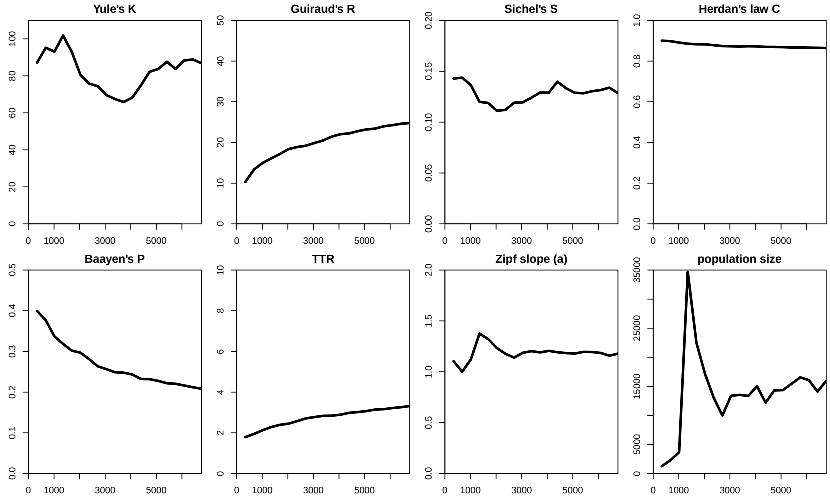


Are these “lexical constants” really constant?



Outline

interactive demo

Outline

Bootstrapping

- An empirical approach to sampling variation:
  - take many random samples from the same population
  - analyse distribution e.g. of productivity measures (mean, median, s.d., boxplot, histogram, ...)
  - alternatively, estimate LNRE model from each sample and analyse distribution of model parameters (➜ later)
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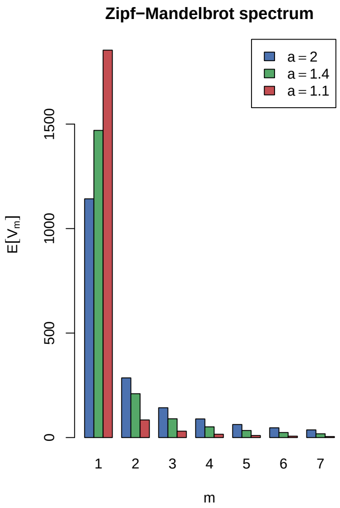
- Parametric bootstrapping
  - use fitted LNRE model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known

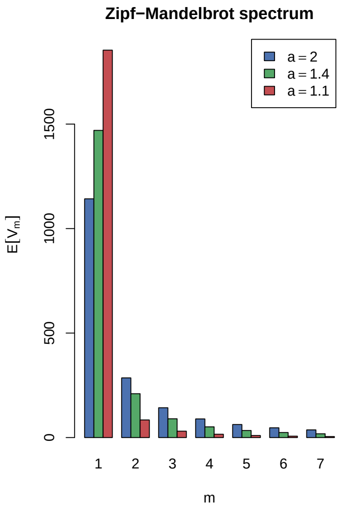
Parametric bootstrapping with LNRE models

- Use simulation experiments to gain better understanding of quantitative measures
  - LNRE model = well-defined population

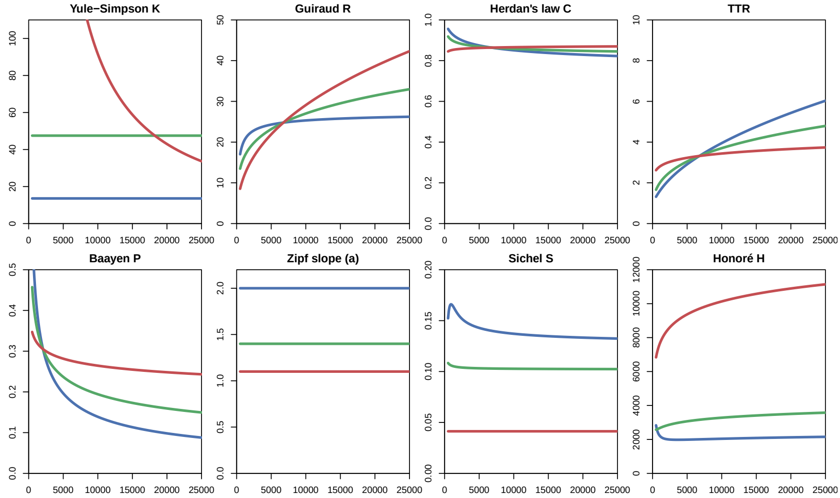


Parametric bootstrapping with LNRE models

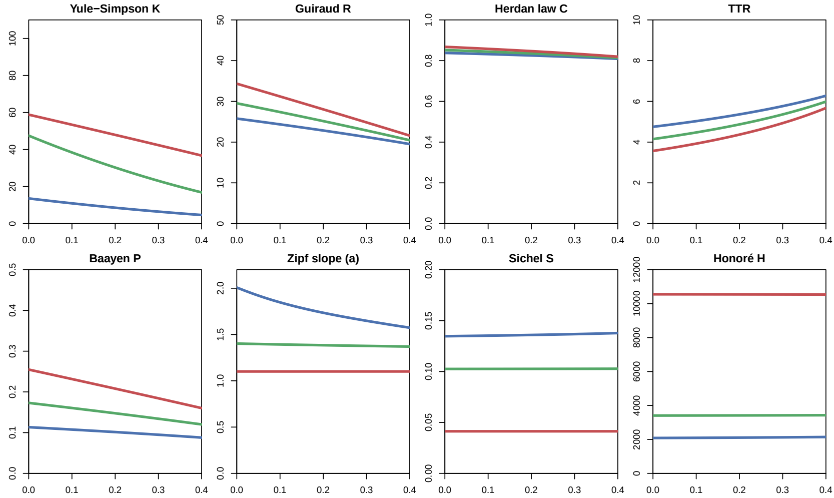
- Use simulation experiments to gain better understanding of quantitative measures
  - LNRE model = well-defined population
- Parametric bootstrapping based on LNRE population
  - dependence on sample size
  - controlled manipulation of confounding factors
  - empirical sampling distribution ➜ variability
- $\mathrm{E}[\mathcal{P}]$ etc. can be computed directly in simple cases



Experiment: sample size

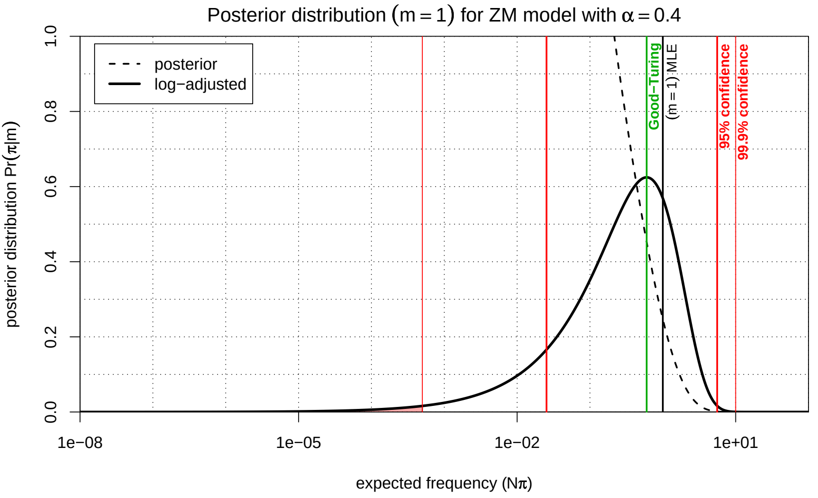


Experiment: frequent lexicalized types



Outline

Posterior distribution



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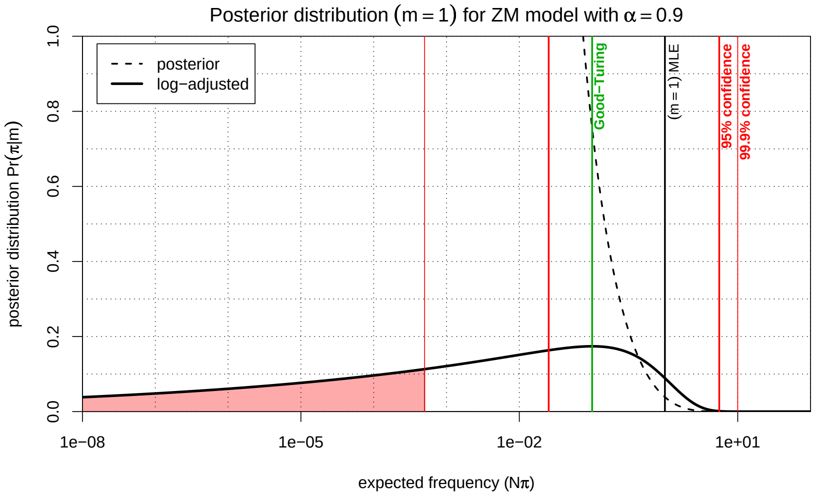

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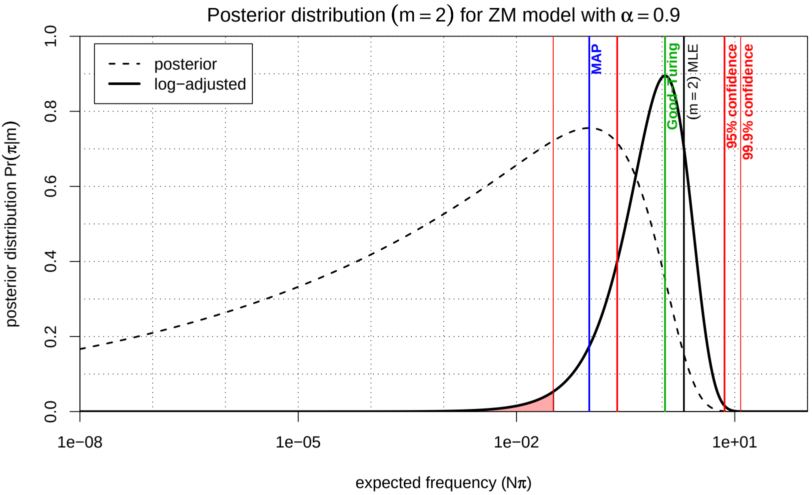

Outline

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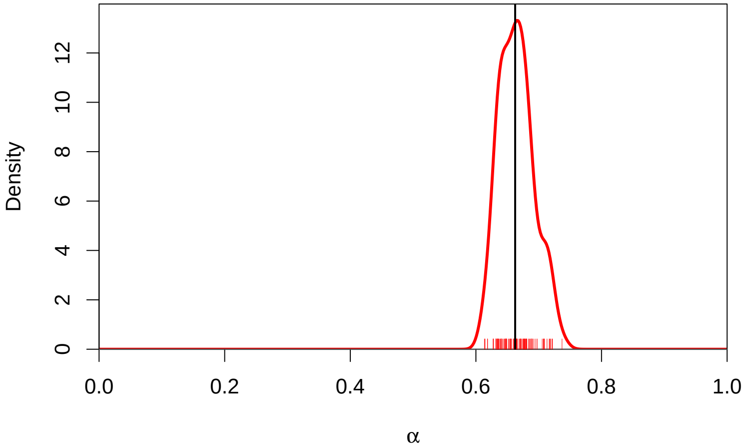
Bootstrapping

parametric bootstrapping with 100 replicates

Zipfian slope $a = 1/\alpha$



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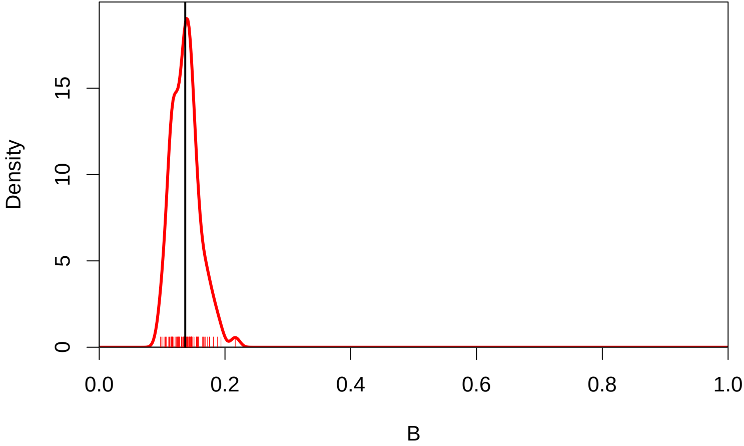
Offset $b = (1 - \alpha)/(B \cdot \alpha)$



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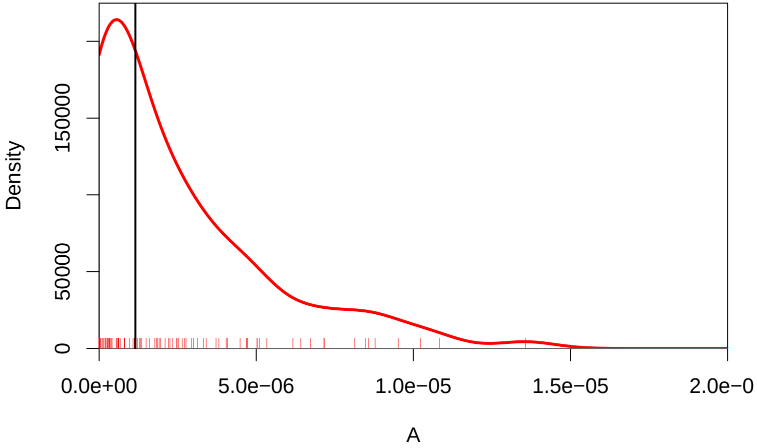
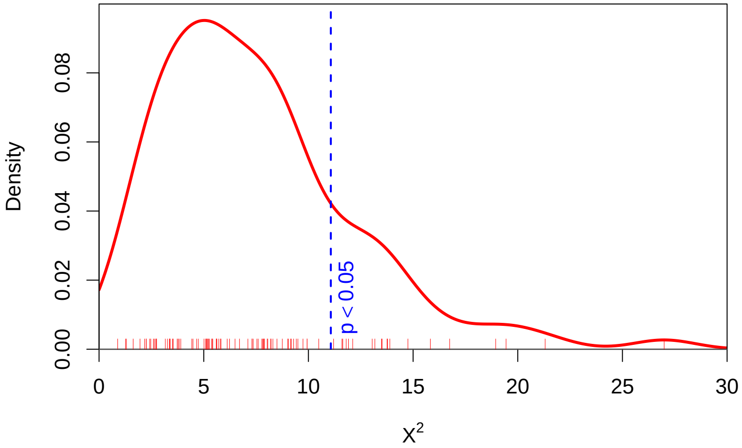
fZM probability cutoff $A = \pi_S$



Bootstrapping

parametric bootstrapping with 100 replicates

Goodness-of-fit statistic X^2 (model not plausible for $X^2 > 11$)



Bootstrapping

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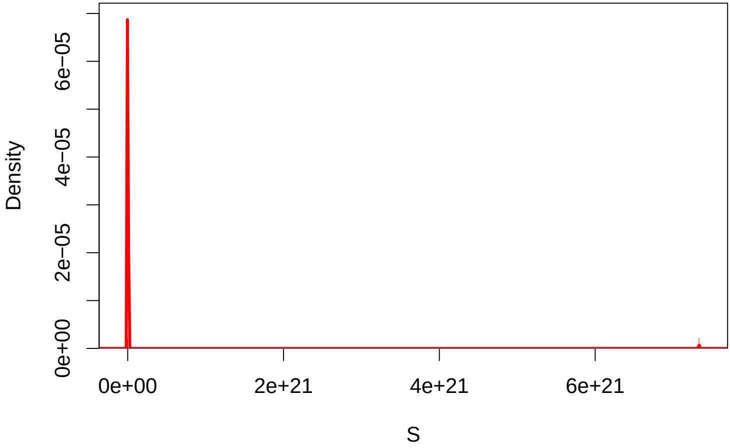
Population diversity S



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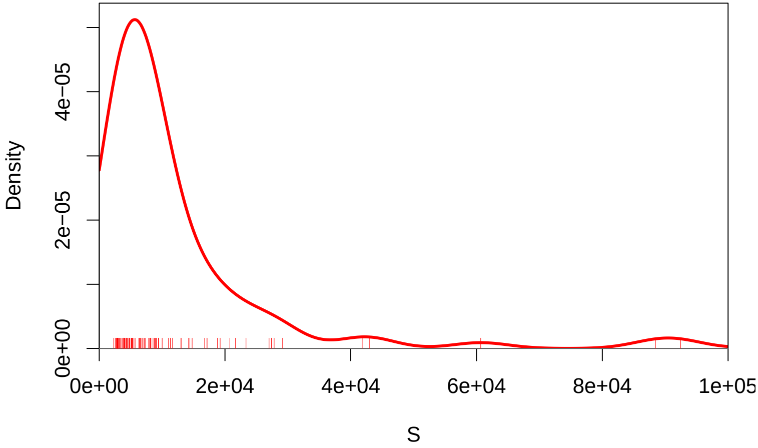
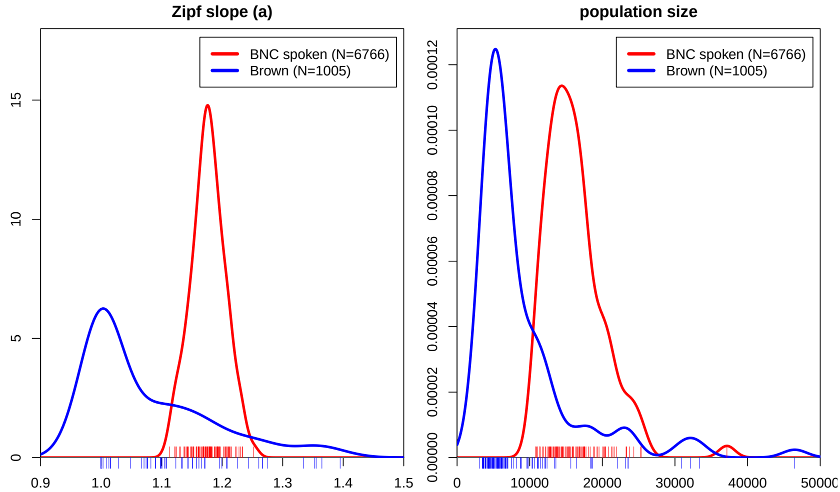


Sample size matters!

Brown corpus is too small for reliable LNRE parameter estimation (bare singulars)



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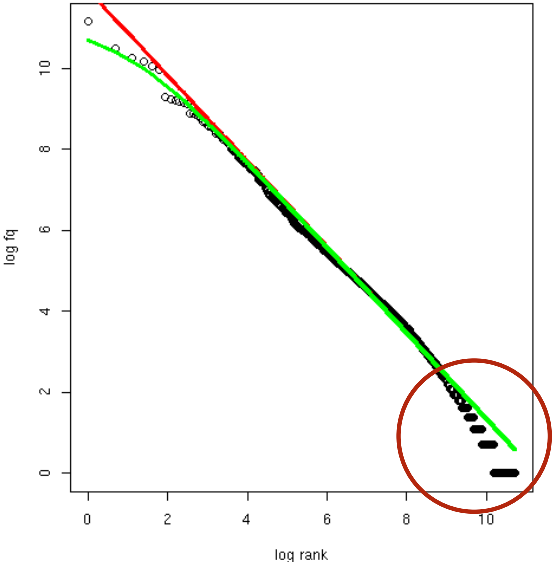
Outline

How well does Zipf's law hold?



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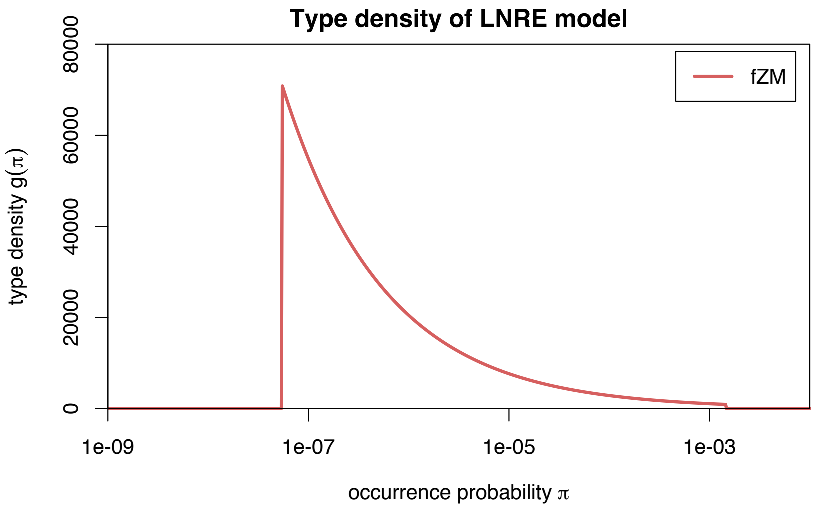
- E.g. Generalized Inverse Gauss-Poisson (GIGP; Sichel 1971)

$$g(\pi) = \frac{(2/bc)^{\gamma+1}}{K_{\gamma+1}(b)} \cdot \pi^{\gamma-1} \cdot e^{-\frac{\pi}{c} - \frac{b^2 c}{4\pi}}$$

The GIGP model (Sichel 1971)



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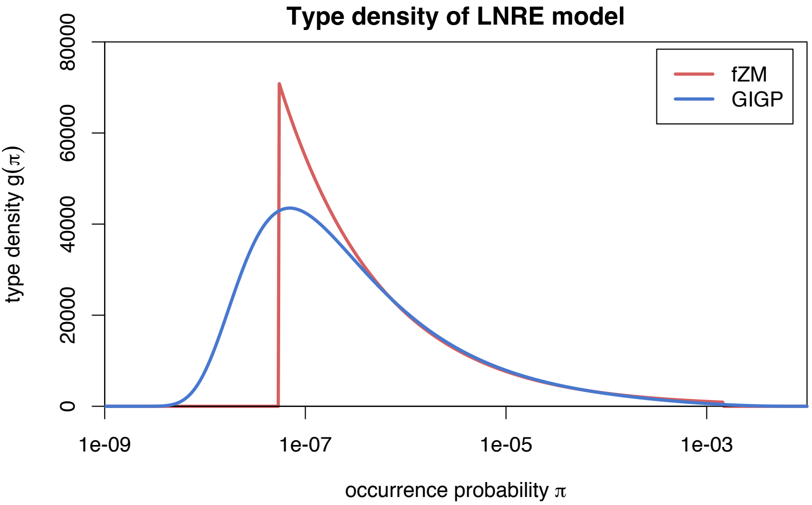
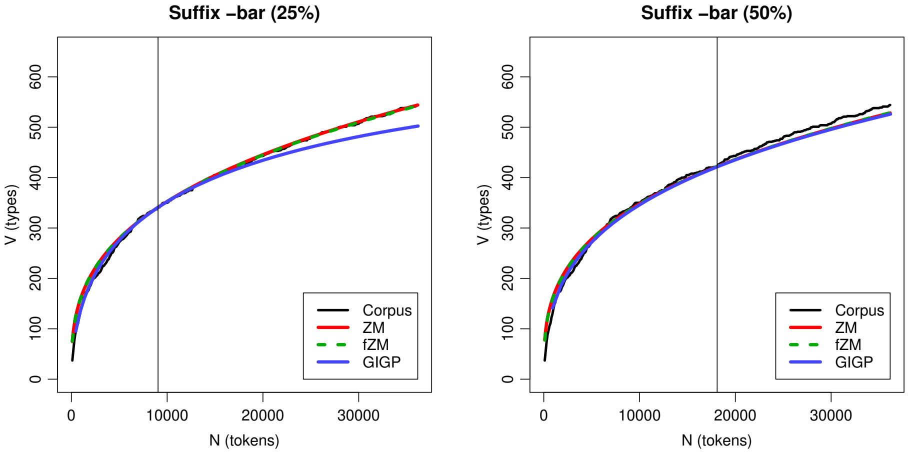


Outline

How accurate is LNRE-based extrapolation?
(Baroni and Evert 2005)



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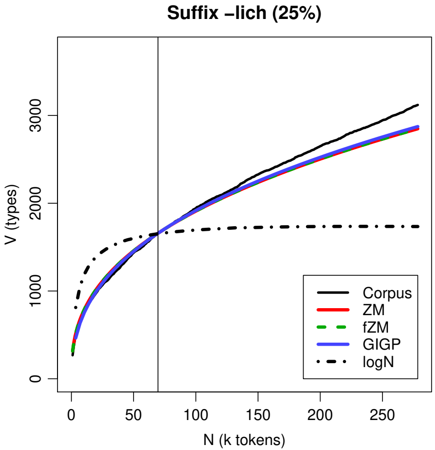

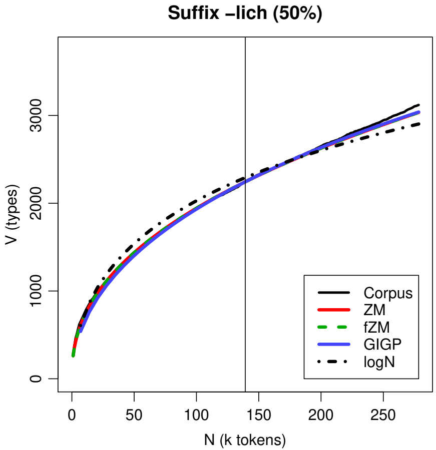

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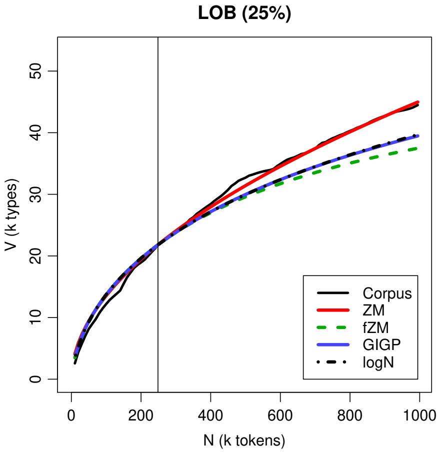

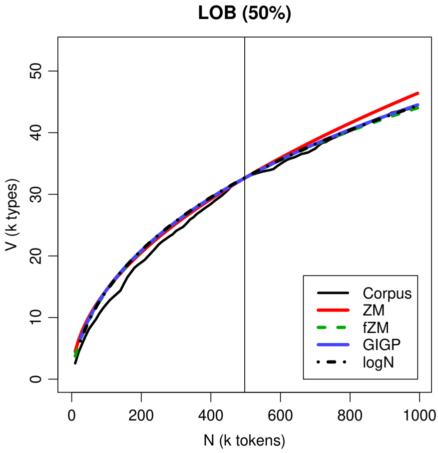

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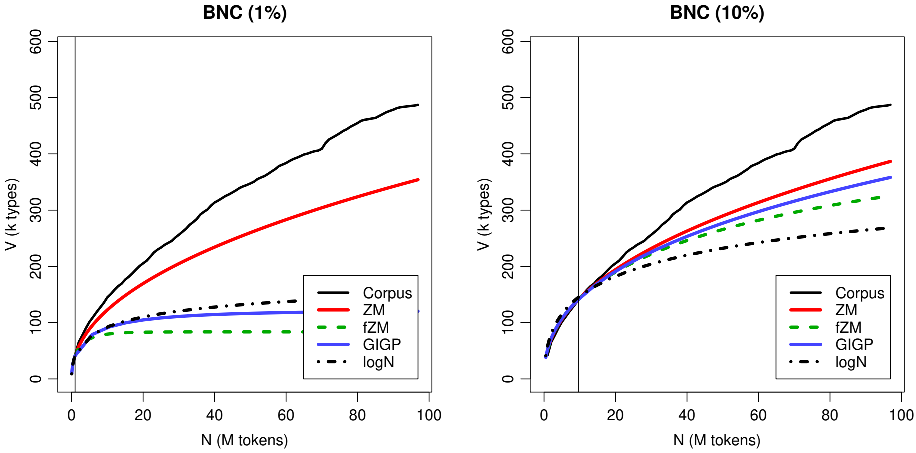

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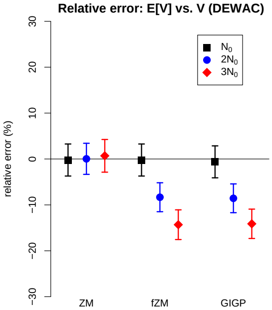
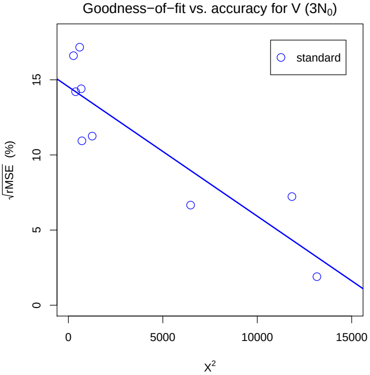
- Cause 2: **non-homogeneous** corpus
  - cannot extrapolate from spoken BNC to written BNC
  - similar for different genres and domains
  - also within single text, e.g. beginning/end of novel

The ECHO correction
(Baroni and Evert 2007)

- Empirical study: quality of extrapolation $N_0 \to 4N_0$ starting from random samples of corpus texts





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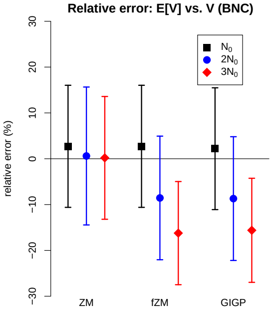

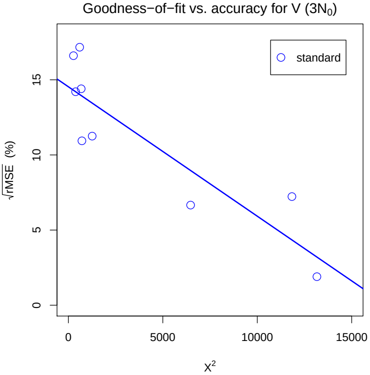

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- Assumption: repetition of type within short span is not a new lexical access or spontaneous formation

A fine example. A very fine example. Only the finest examples. The examples are fine. ...

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- Assumption: repetition of type within short span is not a new lexical access or spontaneous formation
- Replace every repetition within span by special type ECHO
  - N, V and V_1 are not affected ➜ same VGC and $\mathcal{P}$
  - ECHO correction as pre-processing step ➜ no modifications to LNRE models or other analysis software needed

A fine example. ECHO very ECHO ECHO. Only the ECHO ECHO. ECHO ECHO are ECHO. . . .

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- What is an appropriate span size?
  Repetition within textual unit (➜ document frequencies)

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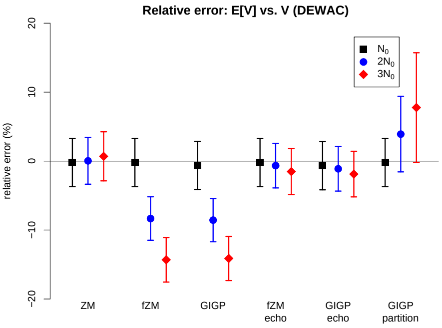

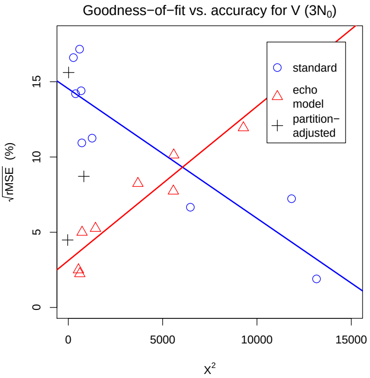

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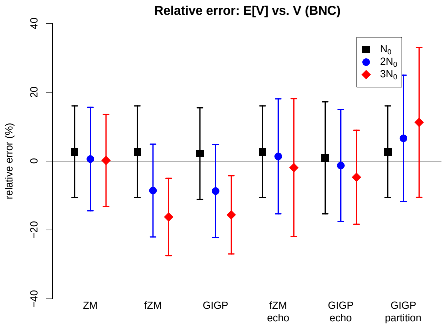

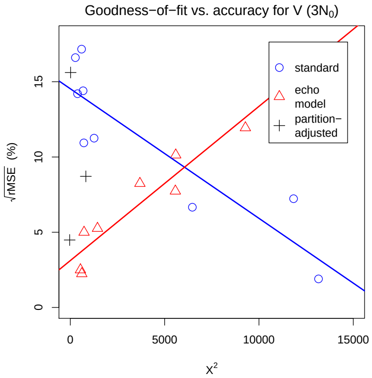

Outline

Case study: Iris Murdoch & early symptoms of AD

(Evert *et al.* 2017)

- Renowned British author (1919–1999)
- Published a total of 26 novels, mostly well received by critics
- Murdoch experienced unexpected difficulties composing her last novel, received “without enthusiasm” (Garrard *et al.* 2005)
- Diagnosis of Alzheimer’s disease shortly after publication

Murdoch novel reveals Alzheimer's

The last novel by the author Iris Murdoch reveals the first signs of Alzheimer's disease, experts say.


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A team from University College London say their examination of works from throughout Dame Iris's career could be used to help diagnose others.

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The study is published online by the journal Brain.

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Conflicting results:

- Decline of lexical diversity in last novel (Garrard *et al.* 2005; Pakhomov *et al.* 2011)
- No clear effects found (Le *et al.* 2011)

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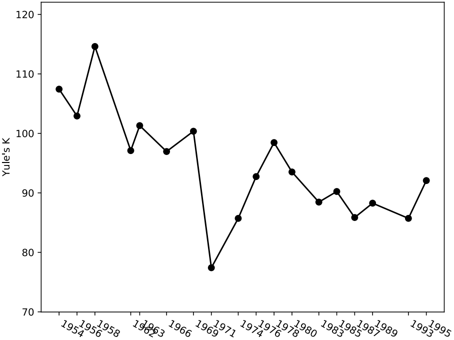
Case study: Iris Murdoch & early symptoms of AD
(Evert *et al.* 2017)

- Corpus data
  - 19 out of 26 novels written by Iris Murdoch
  - including 9 last novels, spanning a period of almost 20 years
  - acquired as e-books (no errors due to OCR)
- Pre-processing and annotation
  - Stanford CoreNLP (Manning *et al.* 2014) for tokenization, sentence splitting, POS tagging, and syntactic parsing
  - exclude dialogue based on typographic quotation marks (following Garrard *et al.* 2005; Pakhomov *et al.* 2011)
- The challenge
  - ☞ assess significance of differences in productivity for single texts
  - ☞ might explain conflicting results in prior work

Measures of vocabulary diversity = productivity
(Evert *et al.* 2017)



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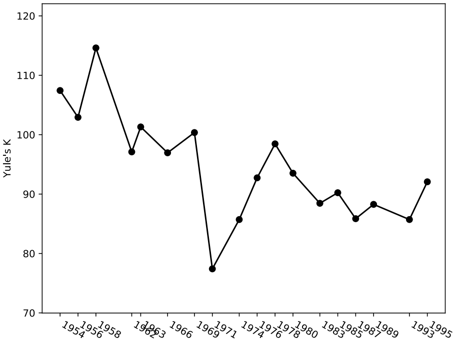

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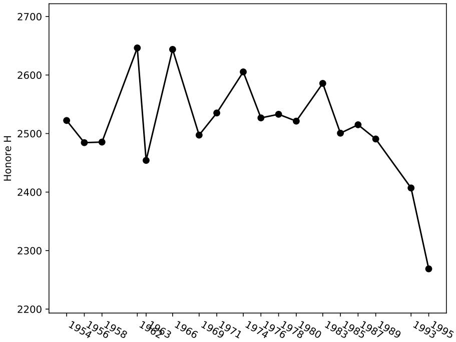

Honoré H

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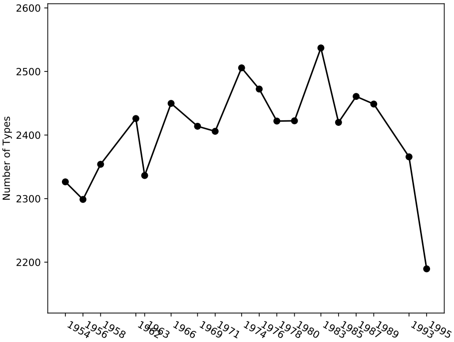
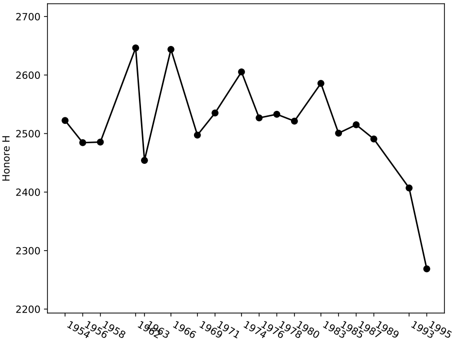


type count / TTR



Honoré H

Cross-validation for productivity measures
(Evert *et al.* 2017)

As a first step:

- Partition each novel into folds of 10,000 consecutive tokens
- ➥ $k \geq 6$ folds for each novel (leftover tokens discarded)

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$$y_1, \ldots, y_k$$

- Compute macro-average as overall measure for the entire text

$$\bar{y} = \frac{y_1 + \cdots + y_k}{k}$$

- Instead of value x obtained by evaluating measure on full text

Cross-validation for productivity measures
(Evert *et al.* 2017)

Significance testing procedure:

- Standard deviation σ of individual folds estimated from data

$$\sigma^2 \approx s^2 = \frac{1}{k-1}\sum_{i=1}^{k}(y_i - \bar{y})^2$$

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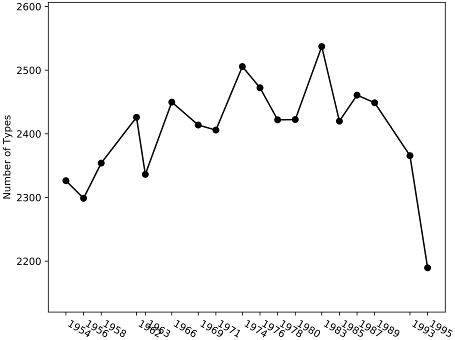
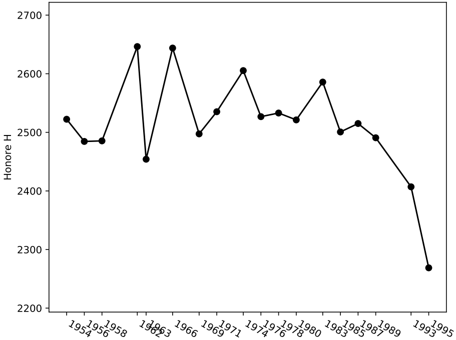
- Comparison of samples with Student's t-test, based on pooled cross-validation folds (feasible even for $n_1 = 1$)

Productivity measures with confidence intervals
(Evert *et al.* 2017)



type count / TTR



Honoré *H*

Productivity measures with confidence intervals

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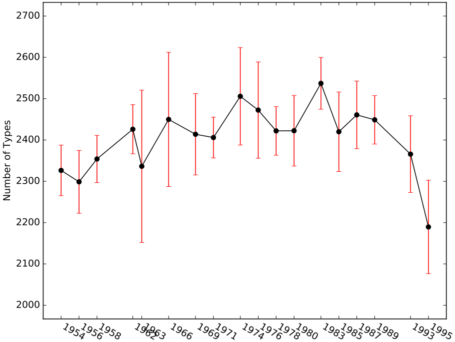

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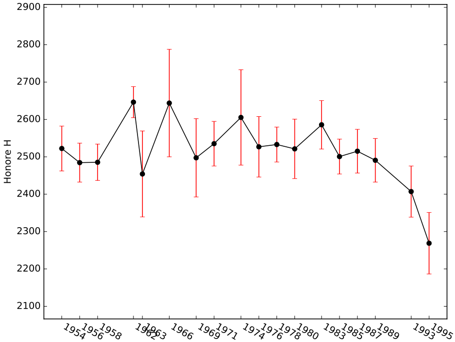

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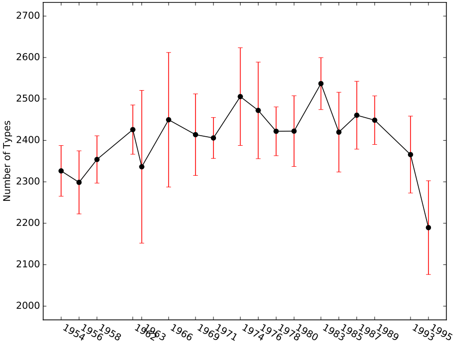

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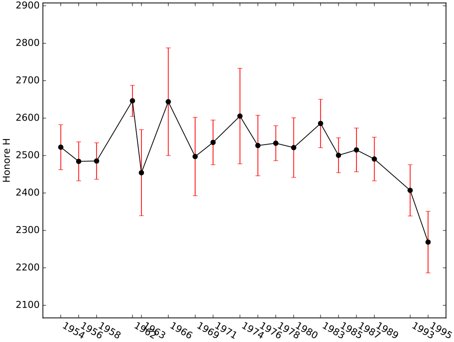
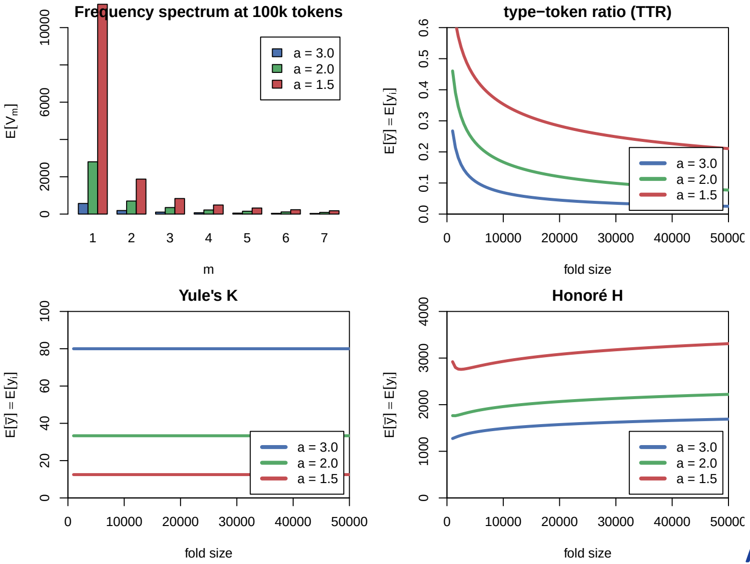


Honoré H

significance test vs. first 17 novels

$t = -6.1$, df=5.52, $p = .0012$**

Cross-validated measures depend on fold size!



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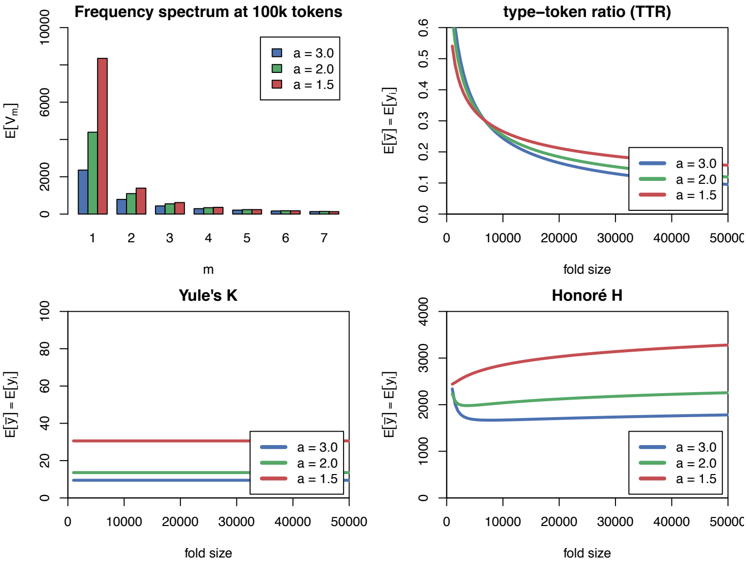

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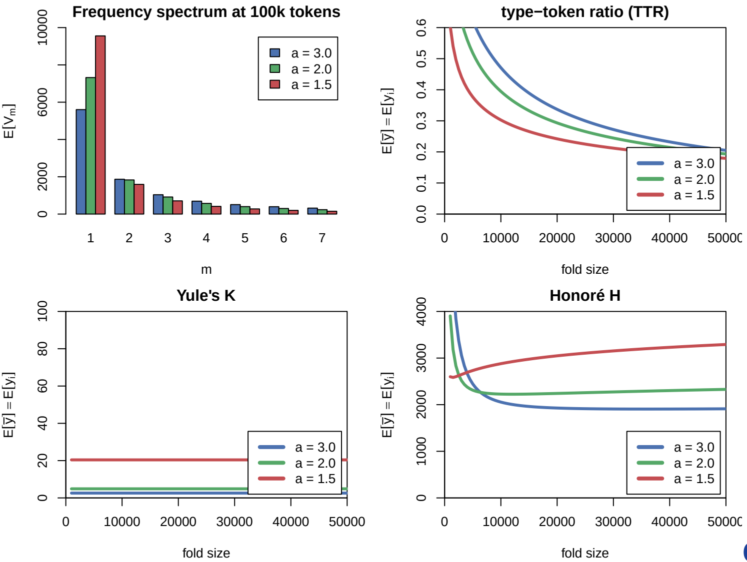

Outline

Research programme for LNRE models

- Improve efficiency & numerical accuracy of implementation
  - numerical integrals instead of differences of Gamma functions
  - better parameter estimation (gradient, aggregated spectrum)
- Analyze accuracy of LNRE approximations
  - comprehensive simulation experiments, esp. for small samples
- Specify more flexible LNRE population models
  - my favourite: piecewise Zipfian type density functions
  - Baayen (2001): mixture distributions (different parameters)
- Develop hypothesis tests & confidence intervals
  - key challenge: goodness-of-fit vs. confidence region
  - prediction intervals for model-based extrapolation
- Simulation experiments for productivity measures
  - Can we find a quantitative measure that is robust against confounding factors and corresponds to intuitive notions of productivity & lexical diversity?

Research programme for LNRE models

- Is non-randomness a problem?
  - not for morphological productivity ➜ ECHO correction
  - tricky to include explicitly in LNRE approach
- Do we need LNRE models for practical applications?
  - better productivity measures + empirical sampling variation
  - based on cross-validation approach (Evert *et al.* 2017)
- How important is semantics & context?
  - Does it make sense to measure productivity and lexical diversity purely in terms of type-token distributions?
  - e.g. register variation for morphological productivity
  - e.g. semantic preferences in productive slots of construction
  - type-token ratio $\neq$ complexity of author's vocabulary

Thank you!

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