Roadmap

Lexical statistics: the basics

Zipf's law
- Typical frequency patterns
- Zipf's law
- Consequences

Applications
- Productivity in morphology
- Productivity beyond morphology
- Lexical richness
- Conclusion and outlook

Counting Words:
Introduction

Marco Baroni & Stefan Evert

Málaga, 7 August 2006

Roadmap

- Introduction and motivation

Roadmap

- Introduction and motivation
- LNRE modeling: soft

Roadmap

- Introduction and motivation
- LNRE modeling: soft
- LNRE modeling: hard

Roadmap

- Introduction and motivation
- LNRE modeling: soft
- LNRE modeling: hard
- Playtime!

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- Introduction and motivation
- LNRE modeling: soft
- LNRE modeling: hard
- Playtime!
- The bad news and outlook

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Zipf's law

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Lexical statistics

Zipf 1949/1961, Baayen 2001, Evert 2005

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- Statistical study of distribution of **types** (words and other units) in texts
- Different from other categorical data because of extreme richness of types

Basic terminology

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- A sample: `a b b c a a b a`
- N: 8; V: 3; V_1: 1

Rank/frequency profile

- The sample: `a b b c a a b a d`

Rank/frequency profile

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- Frequency list ordered by decreasing frequency

t	f
a	4
b	3
c	1
d	1

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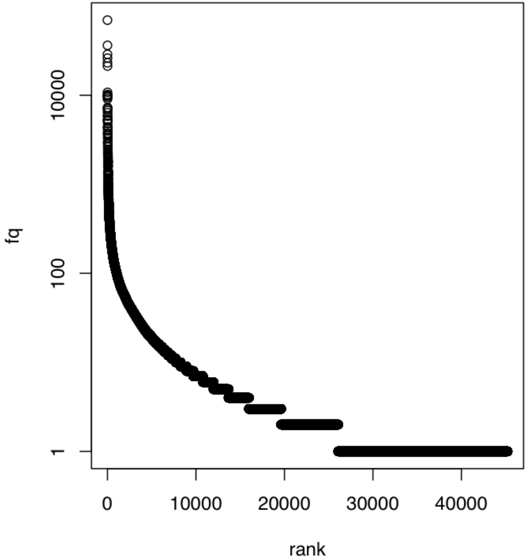
- Allows expression of frequency in function of rank of type

Rank/frequency profile of Brown corpus



Frequency spectrum

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Frequency spectrum

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- Frequency spectrum:

m	V_m
1	2
3	1
4	1

Rank/frequency profiles and frequency spectra

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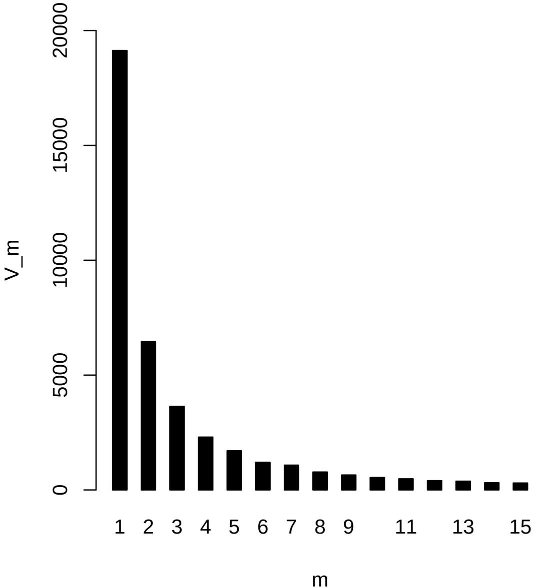
- From rank/frequency profile to spectrum: count occurrences of each f in profile to obtain V_f values of corresponding spectrum elements
- From spectrum to rank/frequency profile: given highest f (i.e., m) in a spectrum, the ranks 1 to V_f in the corresponding rank/frequency profile will have frequency f, the ranks $V_f + 1$ to $V_f + V_g$ (where g is the second highest frequency in the spectrum) will have frequency g, etc.

Frequency spectrum of Brown corpus



Vocabulary growth curve

- The sample: `a b b c a a b a`

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- N: 5, V: 3, V_1: 1

Vocabulary growth curve

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- N: 5, V: 3, V_1: 1
- N: 8, V: 3, V_1: 1

Vocabulary growth curve

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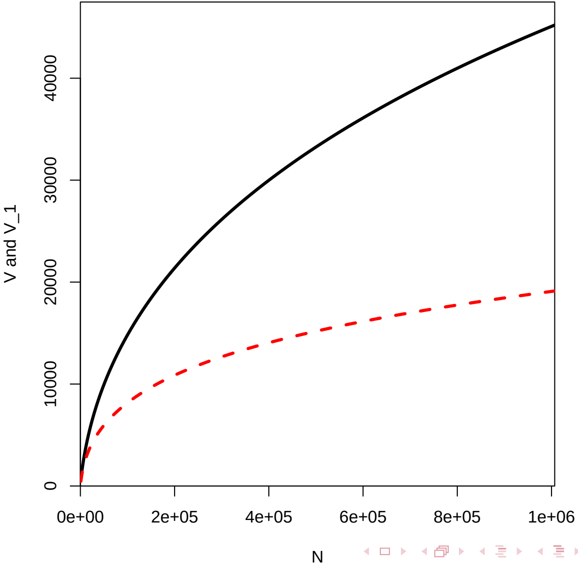
- (Most VGCs on our slides smoothed with **binomial interpolation**)

Vocabulary growth curve of Brown corpus

With V_1 growth in red



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Top and bottom ranks in the Brown corpus

top frequencies			bottom frequencies		
rank	**fq**	**word**	**rank range**	**fq**	**randomly selected examples**
1	62642	the	7967-8522	10	recordings undergone privileges
2	35971	of	8523-9236	9	Leonard indulge creativity
3	27831	and	9237-10042	8	unnatural Lolotte authenticity
4	25608	to	10043-11185	7	diffraction Augusta postpone
5	21883	a	11186-12510	6	uniformly throttle agglutinin
6	19474	in	12511-14369	5	Bud Councilman immoral
7	10292	that	14370-16938	4	verification gleamed groin
8	10026	is	16939-21076	3	Princes nonspecifically Arger
9	9887	was	21077-28701	2	blitz pertinence arson
10	8811	for	28702-53076	1	Salaries Evensen parentheses

Typical frequency patterns
BNC

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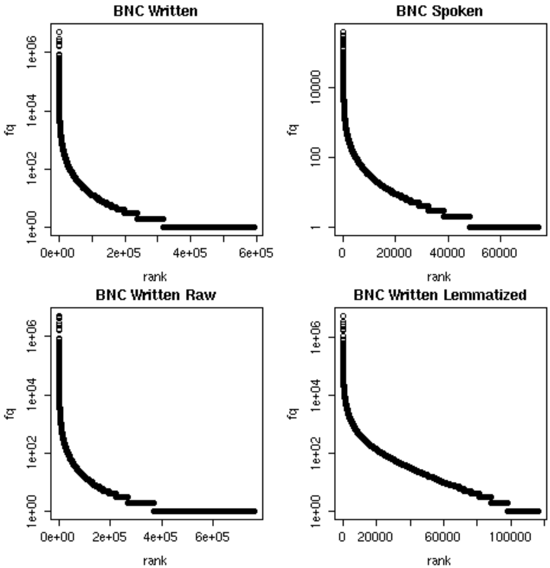

Typical frequency patterns
Other corpora

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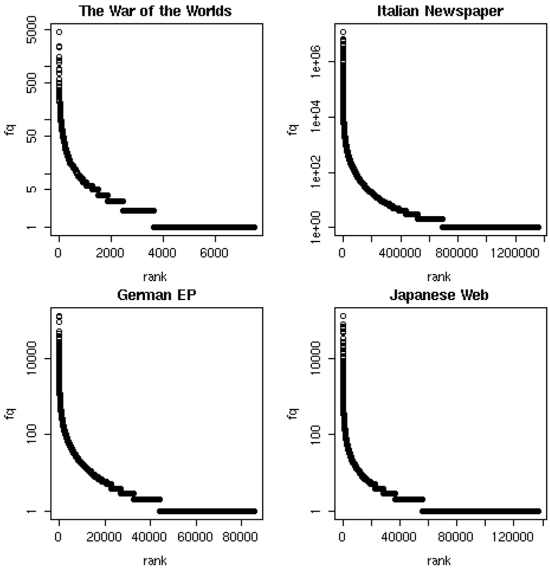
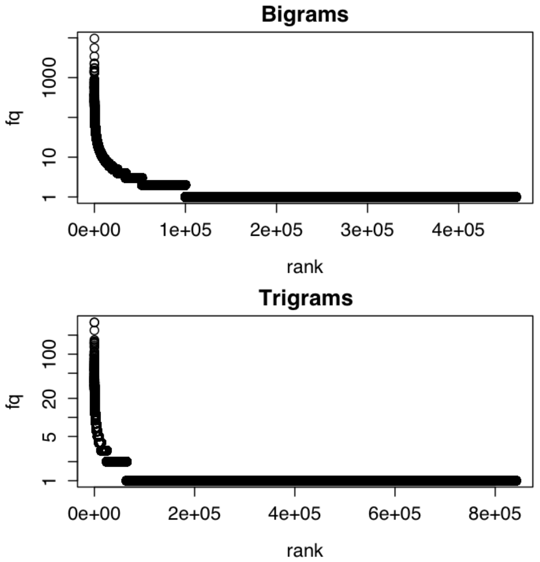
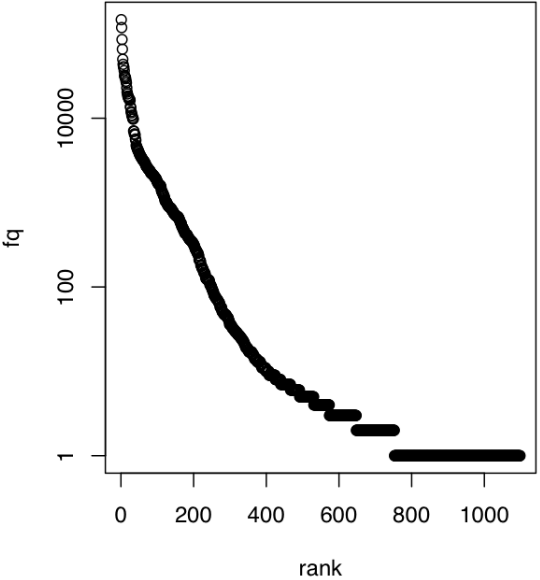


Typical frequency patterns

Brown bigrams and trigrams



Typical frequency patterns
The Italian prefix *ri-* in the *la Repubblica* corpus



Zipf's law

- Language after language, corpus after corpus, linguistic type after linguistic type. . .

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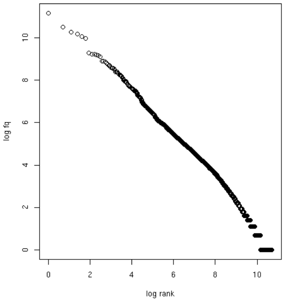

Zipf's law

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- Best fit a and C can be found with least squares method
- Provides intuitive interpretation of a and C:
  - a is **slope** determining how fast log frequency decreases with log rank
  - $\log C$ is **intercept**, i.e., predicted log frequency of word with rank 1 (log rank 0), i.e., most frequent word

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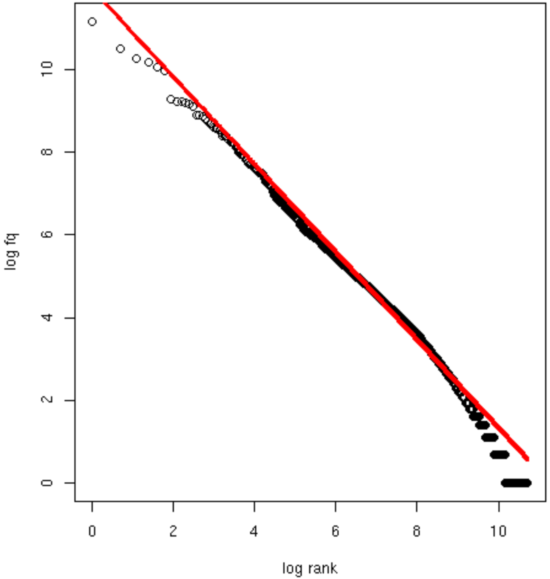

Fit of Zipf's law

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Fit of Zipf's law

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Fit of Zipf's law

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Zipf-Mandelbrot's law
Mandelbrot 1953

- Mandelbrot's extra parameter:

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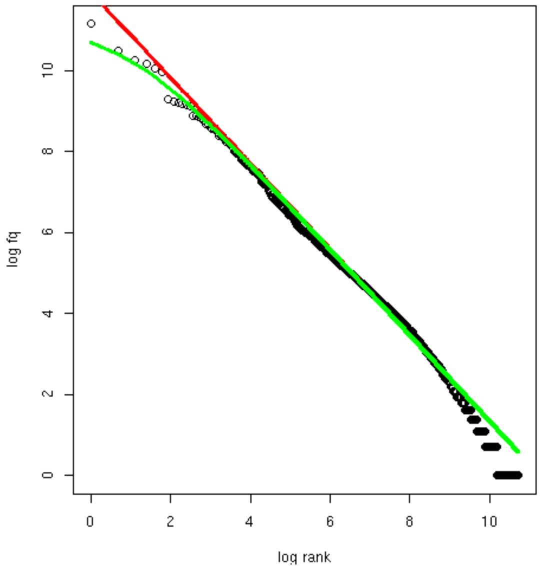
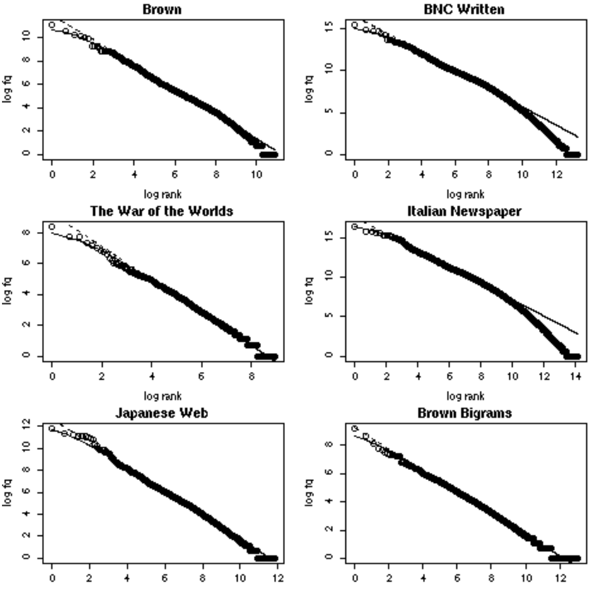
- Zipf-Mandelbrot's law is basis of LNRE statistical models we will introduce

Mandelbrot's adjustment
Fitting the Brown rank/frequency profile



More fits


Brown
BNC Written
The War of the Worlds
Italian Newspaper
Japanese Web
Brown Bigrams
log fq
log rank

A few mildly interesting things about Zipf(-Mandelbrot)'s law

- a is often close to 1 for word frequency distributions (hence simplified version: $f = C/r$, and -1 slope in log-log space)

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- Monkey languages display Zipf's law (intuition: few short words have very high chances to be generated; long tail of highly unlikely long words)

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- Monkey languages display Zipf's law (intuition: few short words have very high chances to be generated; long tail of highly unlikely long words)
- Zipf's law is everywhere (Li 2002)

Consequences

- Data sparseness

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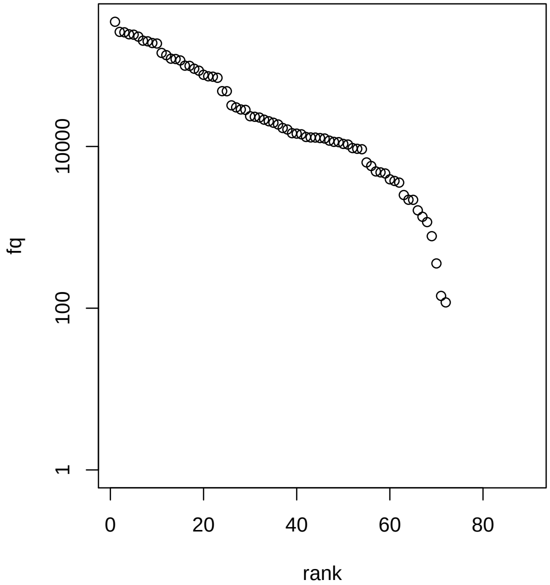
V, sample size and the Zipfian distribution

- Significant tail of hapax legomena indicates that chances of encountering new type if we keep sampling are high
- Zipfian distribution implies vocabulary curve that is still growing at largest sample size

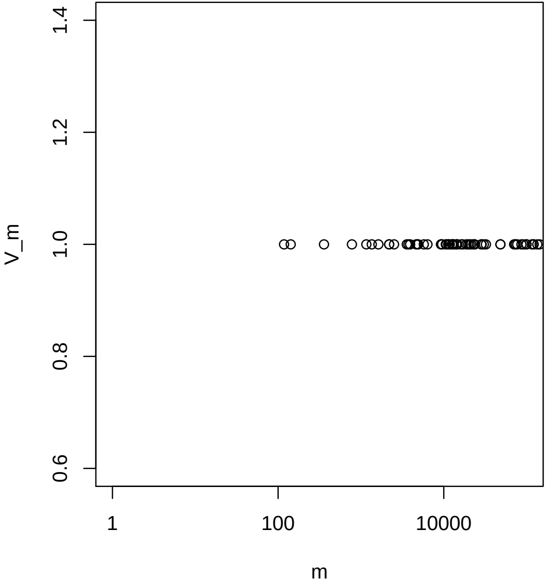
Pronouns in Italian (*la Repubblica*)

Rank/frequency profile



Pronouns in Italian

Frequency spectrum



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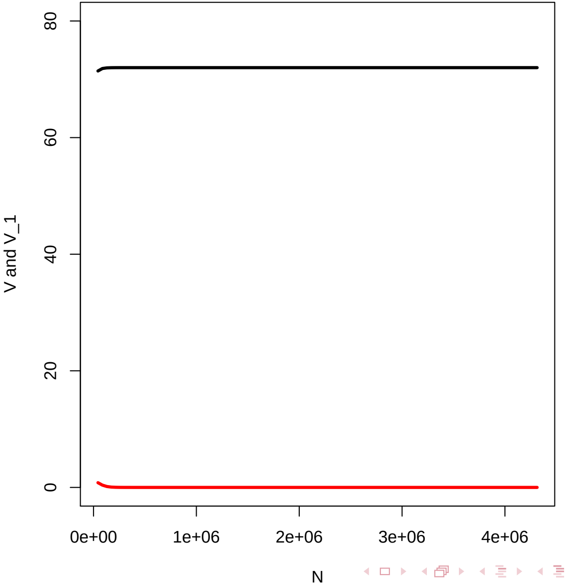

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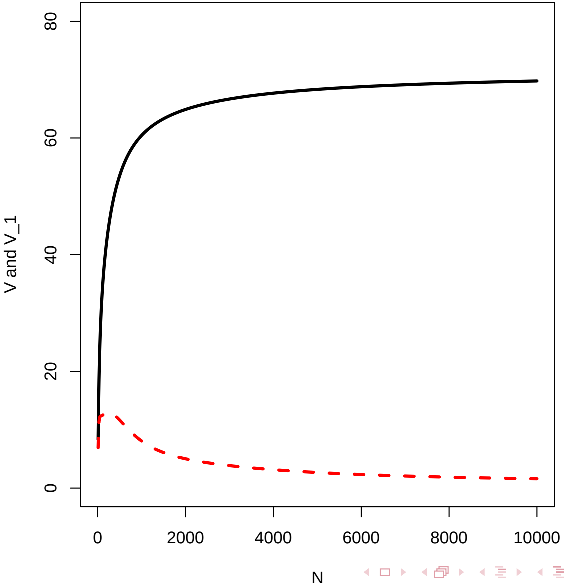
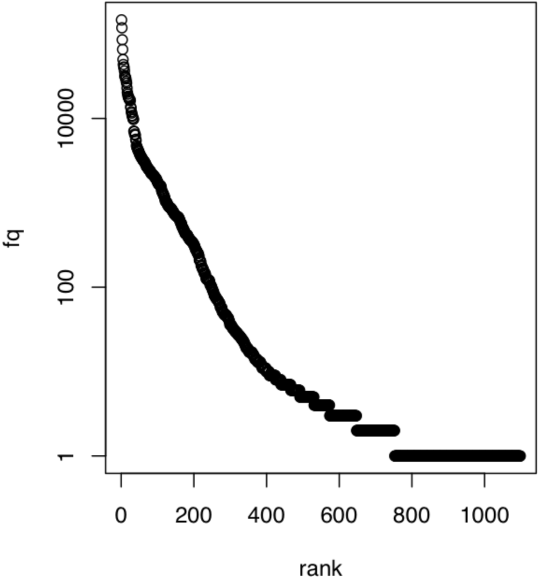


ri- in Italian (*la Repubblica*)
Rank/frequency profile



ri- in Italian
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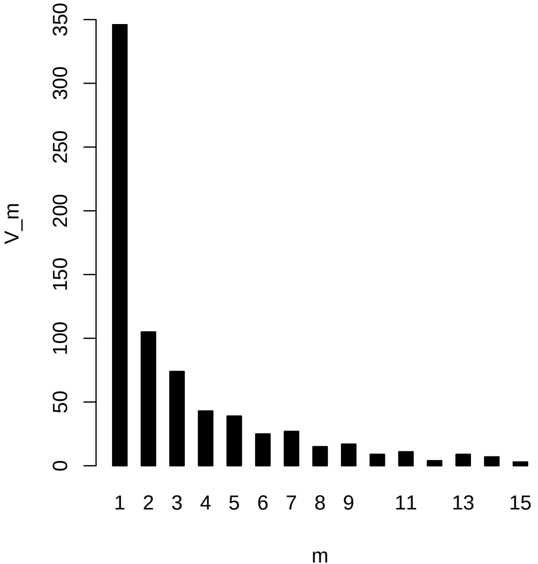
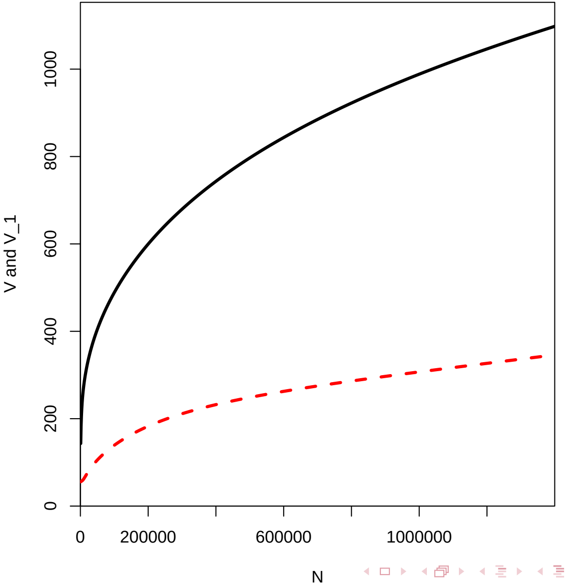


ri- in Italian

Vocabulary growth curve



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Applications

- Productivity (in morphology and elsewhere)
- Lexical richness (in stylometry, language acquisition/pathology and elsewhere)
- Extrapolation of type counts and type frequency distribution for practical NLP purposes (e.g., estimating proportion of OOV words, typos, etc.)
- ... (e.g., Good-Turing smoothing, prior distribution for Bayesian language modeling)

Productivity

- In many linguistic problems, rate of growth of VGC is interesting issue in itself
- Baayen (1989 and later) makes link between linguistic notion of productivity and vocabulary growth rate

Productivity in morphology: the classic definition

Schultink (1961), translated by Booij

Productivity as morphological phenomenon is the possibility which language users have to form an in principle uncountable number of new words unintentionally, by means of a morphological process which is the basis of the form-meaning correspondence of some words they know.

V as a measure of productivity

- Comparable for same N only!

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V as a measure of productivity

- Comparable for same N only!
- Good first approximation, but it is measuring attestedness, not potential:
  - (According to rough BNC counts) *de-* verbs have V of 141, *un-* verbs have V of 119, contra our intuition

V as a measure of productivity

- Comparable for same N only!
- Good first approximation, but it is measuring attestedness, not potential:
  - (According to rough BNC counts) *de-* verbs have V of 141, *un-* verbs have V of 119, contra our intuition
  - We want productivity index of pronouns to be 0, not 72!

Baayen's $\mathscr{P}$

- Operationalize **productivity** of a process as probability that the next token created by the process that we sample is a new word
- This is same as probability that next token in sample is hapax legomenon
- Thus, we can estimate probability of sampling a new word as relative frequency of hapax legomena in our sample:

$$\mathscr{P} = \frac{V_1}{N}$$

Baayen's $\mathscr{P}$

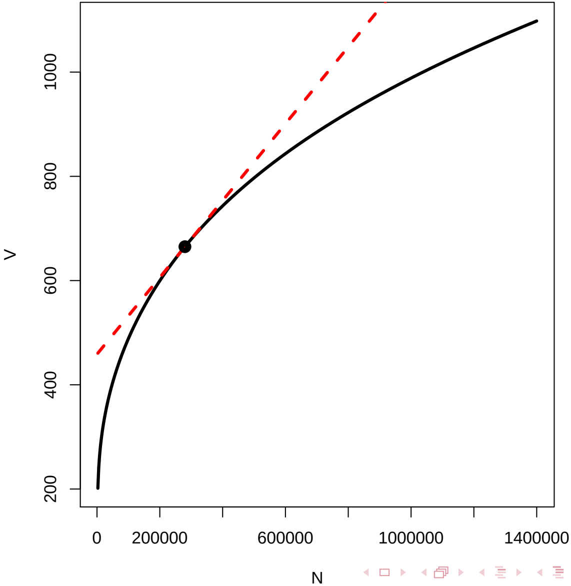
$$\mathscr{P} = \frac{V_1}{N}$$

- Probability to sample token representing type we will never encounter again (token labeled "hapax") at first stage of sampling (when we are at the beginning of N-token-sample) is given by the proportion of hapaxes in the whole N-token-sample divided by the total number of tokens in the sample
- Thus, this must also be probability that *last* token sampled represents new type
- $\mathscr{P}$ as productivity measure matches intuition that productivity should measure *potential* of process to generate new forms

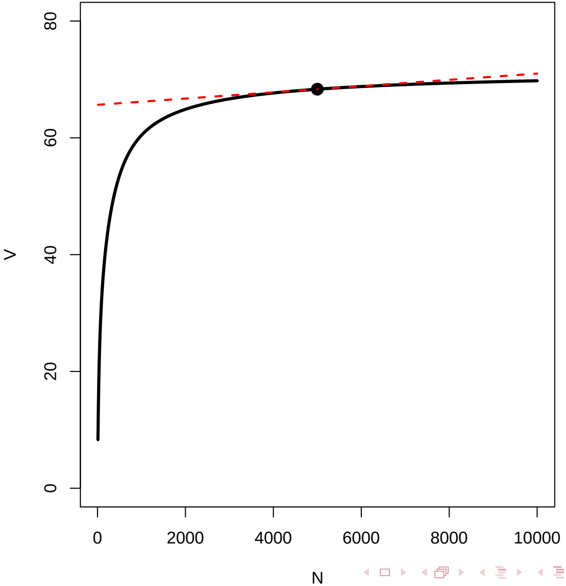
$\mathscr{P}$ as vocabulary growth rate

- $\mathscr{P}$ measures the potentiality of growth of V in a very literal way, i.e., it is the growth rate of V, the rate at which vocabulary size increases
- $\mathscr{P}$ is (approximation to) the *derivative* of V at N, i.e., the slope of the tangent to the vocabulary growth curve at N (Baayen 2001, pp. 49-50)
- Again, "rate of growth" of vocabulary generated by word formation process seems good match for intuition about productivity of word formation process

ri- in Italian *la Repubblica* corpus



Pronouns in Italian *la Repubblica* corpus



Baayen's $\mathscr{P}$ and intuition

class	V	V_1	N	$\mathscr{P}$
it. ri-	1098	346	1,399,898	0.00025
it. pronouns	72	0	4,313,123	0
en. un-	119	25	7,618	.00328
en. de-	141	16	86,130	.000185

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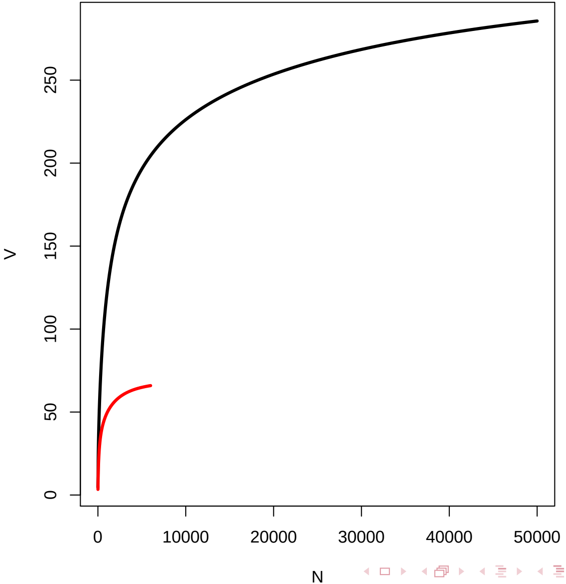
V and *N*
English *re-* and *mis-*



$\mathscr{P}$ and sample size

Introduction

Baroni & Evert

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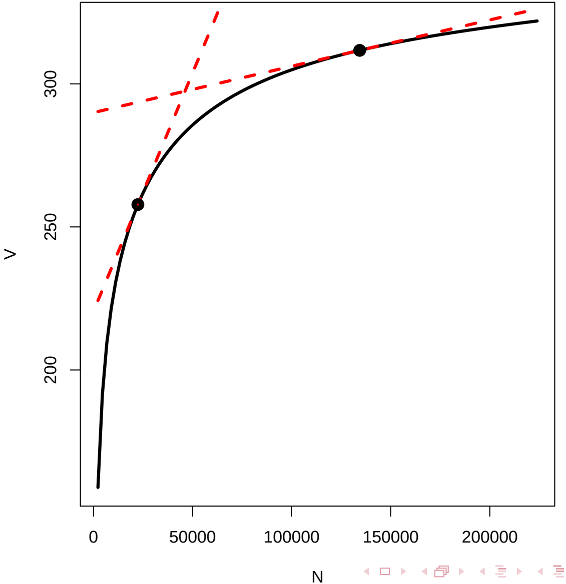
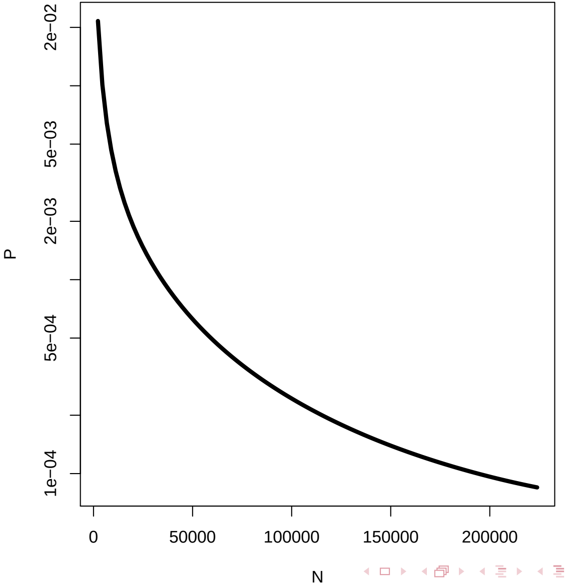
- At the beginning, any word will be a hapax legomenon; as sample increases, hapaxes will be increasingly lower proportion of sample
- A specific instance of the more general problem of "variable constants" (Tweedie and Baayen 1998) in lexical statistics (cf. type/token ratio)

Growth rate of *re-* at different sample sizes



$\mathscr{P}$ as a function of N (*re-*)



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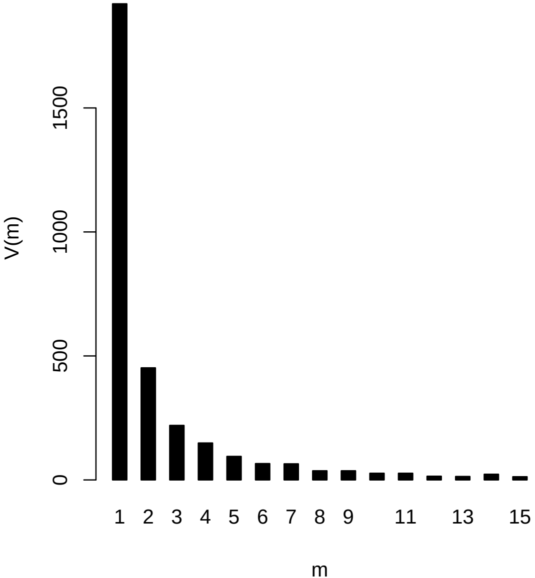
- An example: measuring productivity of NP and PP expansions in German TIGER treebank

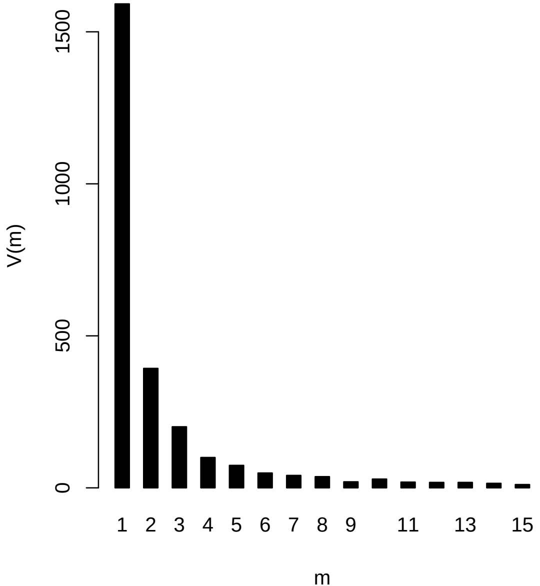
TIGER expansions

- Types are non-terminal rewrite rules for NP and PP, e.g:
  - NP $\longrightarrow$ ART ADJA NN
  - PP $\longrightarrow$ APPR ART NN
- Frequency of occurrence of expansions collected from about 900,000 tokens (50,000 sentences) of German newspaper text from Frankfurter Rundschau
- http://www.ims.uni-stuttgart.de/projekte/TIGER

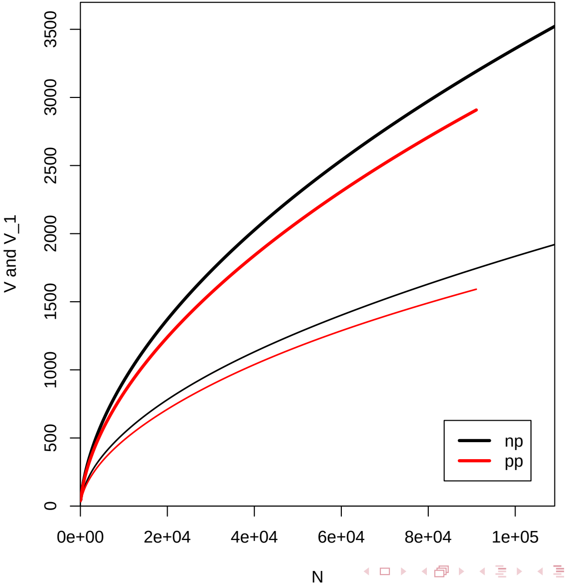
NP spectrum



PP spectrum



Growth curves of NP and PP



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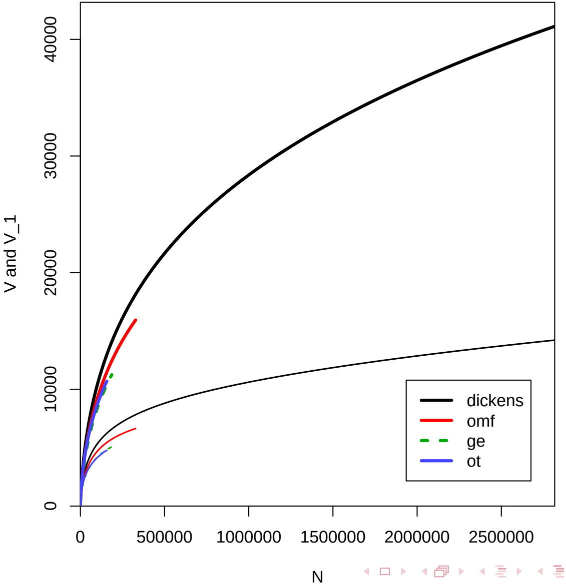
- Are advanced learners distinguishable from native speakers in terms of vocabulary richness? How many words do 5-year-old children know?
- Can changes in V detect the onset of Alzheimer's disease? (Garrard et al. 2005)

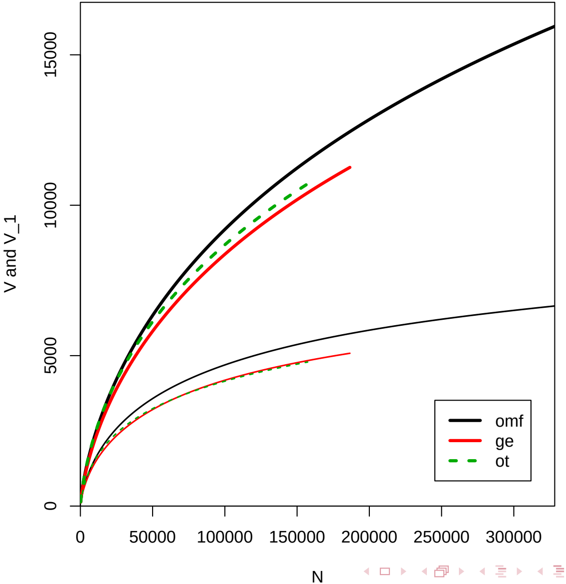
The Dickens' datasets

- Dickens corpus: collection of 14 works by Dickens, about 2.8 million tokens
- Oliver Twist: early work (1837-1839), about 160k tokens
- Great Expectations: later work (1860-1861), considered one of Dickens' masterpieces, about 190k tokens
- Our Mutual Friend: last completed novel (1864-1865), about 330k tokens

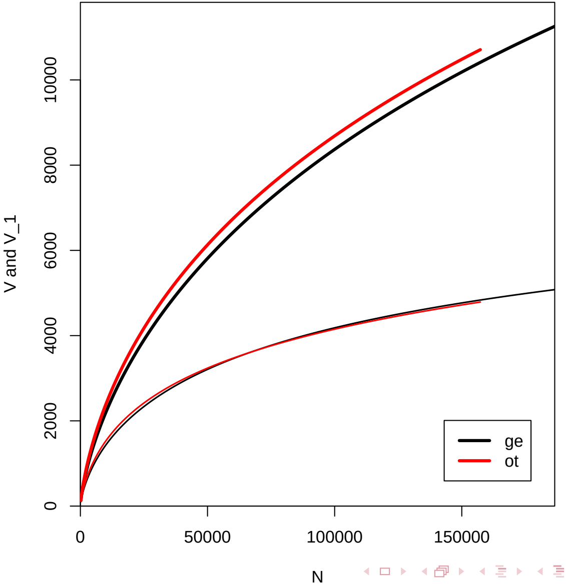
Dickens' V


V and V_1
N
0
10000
20000
30000
40000
0
500000
1000000
1500000
2000000
2500000
dickens
omf
ge
ot

The novels compared



Oliver vs. Great Expectations



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- all applications require a model of the larger **population** of types that our sample comes from
- Two reasons to construct model of type population distribution:
  - Population distribution interesting by itself, for theoretical reasons or in NLP applications
  - We know how to simulate sampling from population; thus once we have population model we can obtain estimates of type-related quantities (e.g., V and V_1) at arbitrary Ns

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Productivity

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- However, type population distribution of word formation process (or other category) might be of interest by itself, as model of a part of the mental lexicon of speaker

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- Lexical richness $= V$ of whole population (how many words did Shakespeare know? Was the lexical repertoire of young Dickens smaller than that of old Dickens? How many words do 5-year-old children know?)

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- Sampling from population, in particular to compute VGC, also of interest

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- Good-Turing estimation, Bayesian priors $\rightarrow$ require full type population model

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