What Every Computational Linguist Should Know About Type-Token Distributions and Zipf's Law

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http://zipfr.r-forge.r-project.org/lrec2018.html

Outline

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Type-token statistics

- Type-token statistics different from most statistical inference
  - not about probability of a specific event
  - but about diversity of events and their probability distribution
- Relatively little work in statistical science
- Nor a major research topic in computational linguistics
  - very specialized, usually plays ancillary role in NLP
- But type-token statistics appear in wide range of applications
  - often crucial for sound analysis

➥ NLP community needs better awareness of statistical techniques, their limitations, and available software

Some research questions

- How many words did Shakespeare know?
- What is the coverage of my treebank grammar on big data?
- How many typos are there on the Internet?
- Is *-ness* more productive than *-ity* in English?
- Are there differences in the productivity of nominal compounds between academic writing and novels?
- Does Dickens use a more complex vocabulary than Rowling?
- Can a decline in lexical complexity predict Alzheimer's disease?
- How frequent is a hapax legomenon from the Brown corpus?
- What is appropriate smoothing for my n-gram model?
- Who wrote the Bixby letter, Lincoln or Hay?
- How many different species of . . . are there? (Brainerd 1982)

Some research questions

-
- coverage estimates
-
-
- productivity
- lexical complexity & stylometry
-
- prior & posterior distribution
-
- unexpected applications
-

Zipf's law (Zipf 1949)

A) Frequency distributions in natural language are highly skewed

B) Curious relationship between rank & frequency

word	r	f	$r \cdot f$
the	1.	142,776	142,776
and	2.	100,637	201,274
be	3.	94,181	282,543
of	4.	74,054	296,216

(Dickens)

C) Various explanations of Zipf's law
  - principle of least effort (Zipf 1949)
  - optimal coding system, MDL (Mandelbrot 1953, 1962)
  - random sequences (Miller 1957; Li 1992; Cao *et al.* 2017)
  - Markov processes ➜ n-gram models (Rouault 1978)

D) Language evolution: birth-death-process (Simon 1955)

☞ not the main topic today!

Outline

Tokens & types

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

type-frequency list

w	f_w
recently	1
very	5
not	3
otherwise	1
much	2
merely	2
now	1

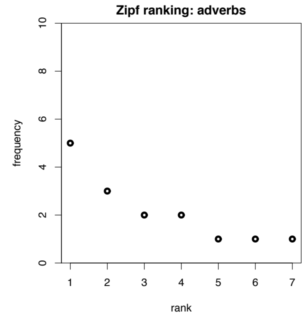
Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

Zipf ranking

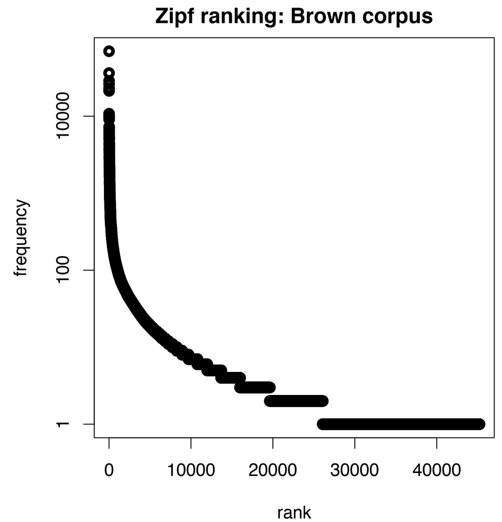
w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1



A realistic Zipf ranking: the Brown corpus

top frequencies			bottom frequencies		
r	f	word	rank range	f	randomly selected examples
1	69836	the	7731 – 8271	10	schedules, polynomials, bleak
2	36365	of	8272 – 8922	9	tolerance, shaved, hymn
3	28826	and	8923 – 9703	8	decreased, abolish, irresistible
4	26126	to	9704 – 10783	7	immunity, cruising, titan
5	23157	a	10784 – 11985	6	geographic, lauro, portrayed
6	21314	in	11986 – 13690	5	grigori, slashing, developer
7	10777	that	13691 – 15991	4	sheath, gaulle, ellipsoids
8	10182	is	15992 – 19627	3	mc, initials, abstracted
9	9968	was	19628 – 26085	2	thar, slackening, deluxe
10	9801	he	26086 – 45215	1	beck, encompasses, second-place

A realistic Zipf ranking: the Brown corpus



Frequency spectrum

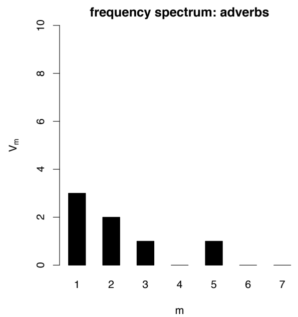
- pool types with $f = 1$ (hapax legomena), types with $f = 2$ (dis legomena), ..., $f = m$, ...
- $V_1 = 3$: number of hapax legomena (*now, otherwise, recently*)
- $V_2 = 2$: number of dis legomena (*merely, much*)
- general definition: $V_m = |\{w \mid f_w = m\}|$

Zipf ranking

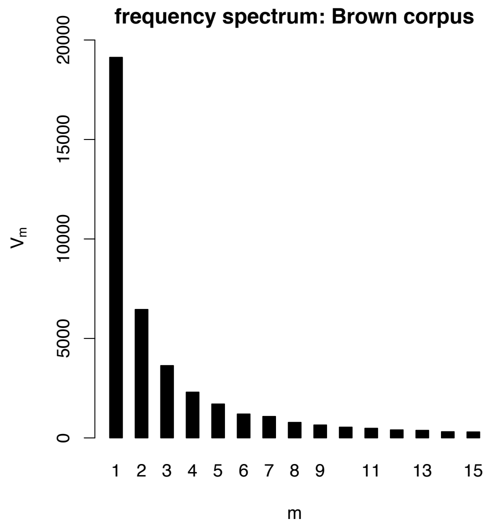
w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1

frequency spectrum

m	V_m
1	3
2	2
3	1
5	1



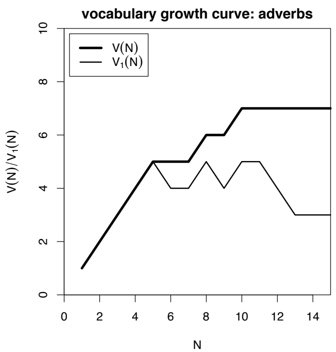
A realistic frequency spectrum: the Brown corpus



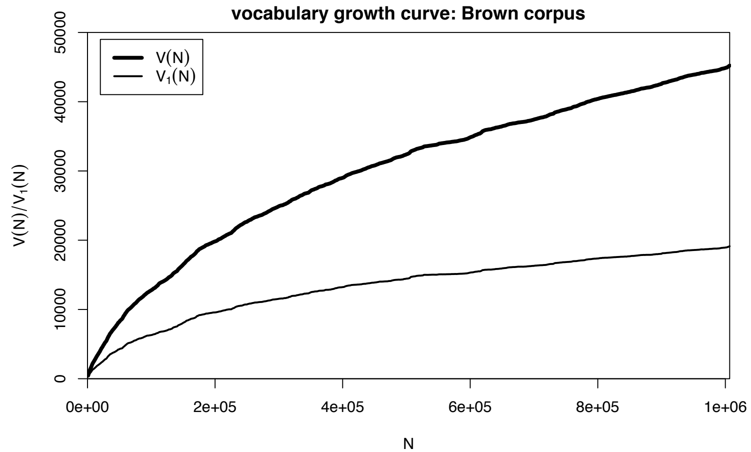
Vocabulary growth curve

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 1,\ V(N) = 1,\ V_1(N) = 1$
- $N = 3,\ V(N) = 3,\ V_1(N) = 3$
- $N = 7,\ V(N) = 5,\ V_1(N) = 4$
- $N = 12,\ V(N) = 7,\ V_1(N) = 4$
- $N = 15,\ V(N) = 7,\ V_1(N) = 3$

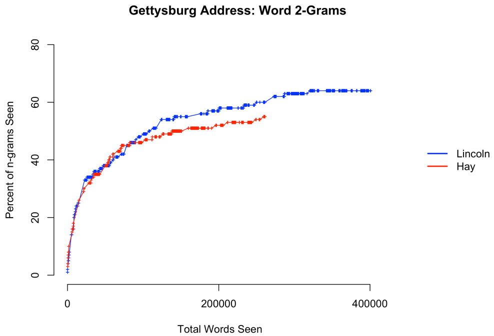


A realistic vocabulary growth curve: the Brown corpus



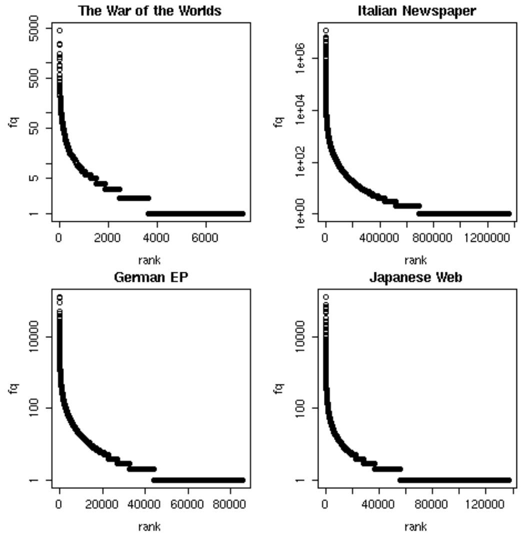
Vocabulary growth in authorship attribution

- Authorship attribution by n-gram tracing applied to the case of the Bixby letter (Grieve *et al.* submitted)
- Word or character n-grams in disputed text are compared against large "training" corpora from candidate authors



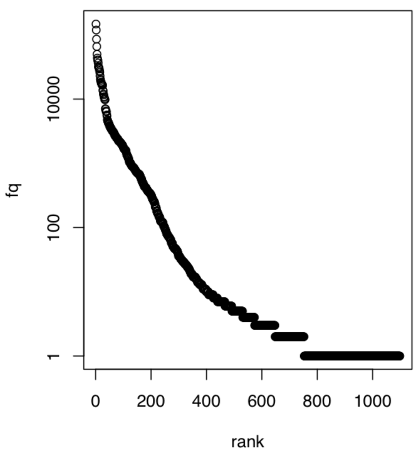
Observing Zipf's law

across languages and different linguistic units

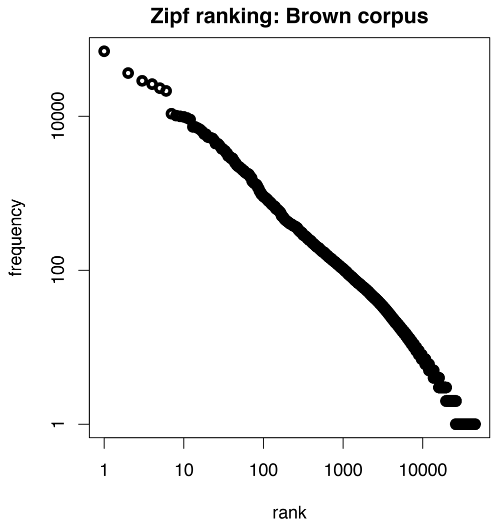


Observing Zipf's law

The Italian prefix *ri-* in the *la Repubblica* corpus



Observing Zipf's law



Observing Zipf's law

- Straight line in double-logarithmic space corresponds to **power law** for original variables
- This leads to Zipf's (1949; 1965) famous law:

$$f_r = \frac{C}{r^a}$$

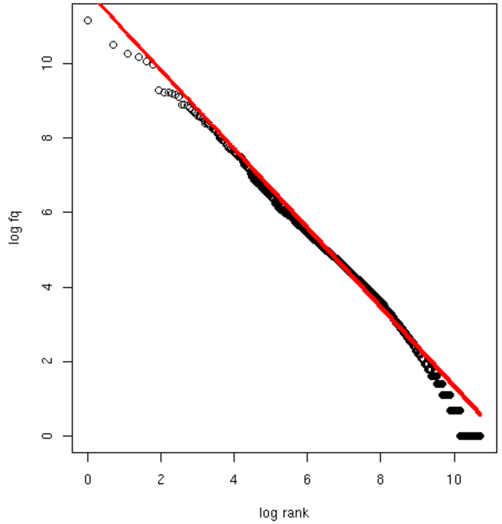
- If we take logarithm on both sides, we obtain:

$$\underbrace{\log f_r}_{y} = \log C - a \cdot \underbrace{\log r}_{x}$$

- Intuitive interpretation of a and C:
  - a is **slope** determining how fast log frequency decreases
  - $\log C$ is **intercept**, i.e. log frequency of most frequent word ($r = 1$ ➜ $\log r = 0$)

Observing Zipf's law

Least-squares fit = linear regression in log-space (Brown corpus)



Zipf-Mandelbrot law

Mandelbrot (1953, 1962)

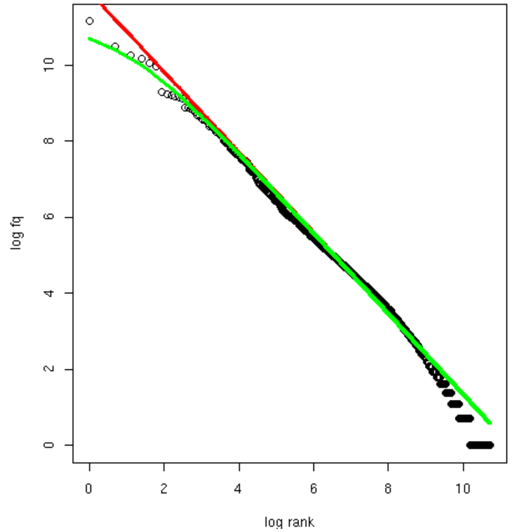
- Mandelbrot's extra parameter:

$$f_r = \frac{C}{(r+b)^a}$$

- Zipf's law is special case with $b = 0$
- Assuming $a = 1$, $C = 60{,}000$, $b = 1$:
  - For word with rank 1, Zipf's law predicts frequency of 60,000; Mandelbrot's variation predicts frequency of 30,000
  - For word with rank 1,000, Zipf's law predicts frequency of 60; Mandelbrot's variation predicts frequency of 59.94
- Zipf-Mandelbrot law forms basis of statistical LNRE models
  - ZM law derived mathematically as limiting distribution of vocabulary generated by a character-level Markov process

Zipf-Mandelbrot law

Non-linear least-squares fit (Brown corpus)



Outline

zipfR
Evert and Baroni (2007)

- `http://zipfR.R-Forge.R-Project.org/`
- Conveniently available from CRAN repository
- Package vignette = gentle tutorial introduction

First steps with zipfR

- Set up a folder for this course, and make sure it is your working directory in R (preferably as an RStudio project)
- Install the most recent version of the zipfR package
- Package, handouts, code samples & data sets available from `http://zipfr.r-forge.r-project.org/lrec2018.html`

```
> library(zipfR)

> ?zipfR   # documentation entry point

> vignette("zipfr-tutorial")   # read the zipfR tutorial
```


Loading type-token data

- Most convenient input: sequence of tokens as text file in vertical format ("one token per line")
  - ☞ mapped to appropriate types: normalized word forms, word pairs, lemmatized, semantic class, n-gram of POS tags, ...
  - ☞ language data should always be in UTF-8 encoding!
  - ☞ large files can be compressed (.gz, .bz2, .xz)
- Sample data: `brown_adverbs.txt` on tutorial homepage
  - lowercased adverb tokens from Brown corpus (original order)
  - ☞ download and save to your working directory

```
> adv <- readLines("brown_adverbs.txt", encoding="UTF-8")

> head(adv, 30)  # mathematically, a ‘‘vector’’ of tokens
> length(adv)    # sample size = 52,037 tokens
```

Descriptive statistics: type-frequency list

```
> adv.tfl <- vec2tfl(adv)
> adv.tfl
    k    f  type
1   1 4859   not
2   2 2084   n't
3   3 1464    so
4   4 1381  only
5   5 1374  then
6   6 1309   now
7   7 1134  even
8   8 1089    as
    :    :     :
     N    V
 52037 1907

> N(adv.tfl)   # sample size
> V(adv.tfl)   # type count
```


Descriptive statistics: frequency spectrum

```
> adv.spc <- tfl2spc(adv.tfl)   # or directly with vec2spc
> adv.spc
    m  Vm
1   1 762
2   2 260
3   3 144
4   4  99
5   5  69
6   6  50
7   7  40
8   8  34
    :   :
     N    V
 52037 1907

> N(adv.spc)   # sample size
> V(adv.spc)   # type count
```


Descriptive statistics: vocabulary growth

- VGC lists vocabulary size $V(N)$ at different sample sizes N
- Optionally also spectrum elements $V_m(N)$ up to `m.max`

```
> adv.vgc <- vec2vgc(adv, m.max=2)
```

- Visualize descriptive statistics with `plot` method

```
> plot(adv.tfl)              # Zipf ranking
> plot(adv.tfl, log="xy")    # logarithmic scale recommended

> plot(adv.spc)              # barplot of frequency spectrum

> plot(adv.vgc, add.m = 1:2) # vocabulary growth curve
```


Further example data sets

`?Brown`	words from Brown corpus
`?BrownSubsets`	various subsets
`?Dickens`	words from novels by Charles Dickens
`?ItaPref`	Italian word-formation prefixes
`?TigerNP`	NP and PP patterns from German Tiger treebank
`?Baayen2001`	frequency spectra from Baayen (2001)
`?EvertLuedeling2001`	German word-formation affixes (manually corrected data from Evert and Lüdeling 2001)

Practice:

- Explore these data sets with descriptive statistics
- Try different plot options (from help pages `?plot.tfl`, `?plot.spc`, `?plot.vgc`)

Outline

Motivation

- Interested in productivity of affix, vocabulary of author, ...; not in a particular text or sample
  - ☞ statistical inference from sample to population
- Discrete frequency counts are difficult to capture with generalizations such as Zipf's law
  - Zipf's law predicts many impossible types with $1 < f_r < 2$
  - ☞ population does not suffer from such quantization effects

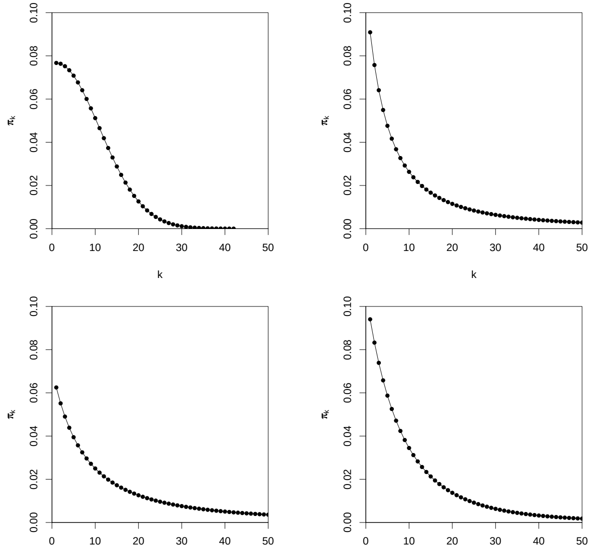
LNRE models

- This tutorial introduces the state-of-the-art LNRE approach proposed by Baayen (2001)
  - LNRE = Large Number of Rare Events
- LNRE uses various approximations and simplifications to obtain a tractable and elegant model
- Of course, we could also estimate the precise discrete distributions using MCMC simulations, but ...
  1. LNRE model usually minor component of complex procedure
  2. often applied to very large samples ($N > 1$ M tokens)

The LNRE population

- Population: set of S types w_i with occurrence **probabilities** π_i
- S = **population diversity** can be finite or infinite ($S = \infty$)
- Not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$
- Need **parametric** statistical **model** to describe full population (esp. for $S = \infty$), i.e. a function $i \mapsto \pi_i$
  - type probabilities π_i cannot be estimated reliably from a sample, but parameters of this function can
  - NB: population index $i \neq$ Zipf rank r

Examples of population models



The Zipf-Mandelbrot law as a population model

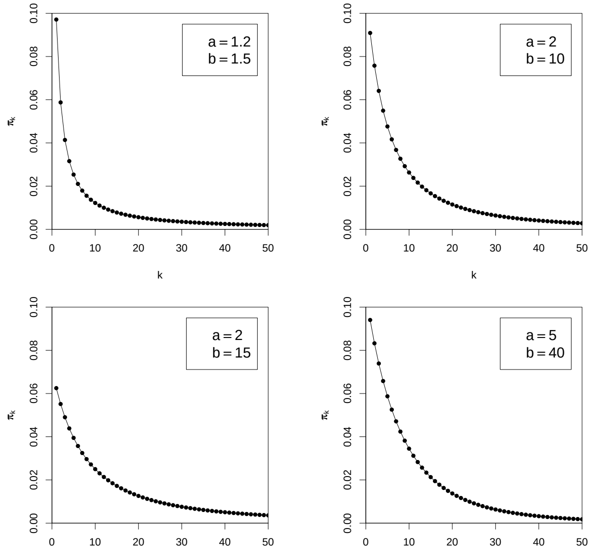
What is the right family of models for lexical frequency distributions?

- We have already seen that the Zipf-Mandelbrot law captures the distribution of observed frequencies very well
- Re-phrase the law for type probabilities:

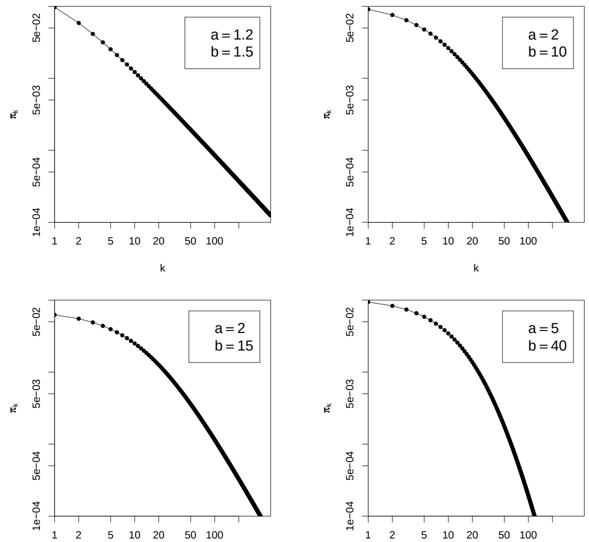
$$\pi_i := \frac{C}{(i+b)^a}$$

- Two free parameters: $a > 1$ and $b \geq 0$
- C is not a parameter but a normalization constant, needed to ensure that $\sum_i \pi_i = 1$
- This is the **Zipf-Mandelbrot** population model

The parameters of the Zipf-Mandelbrot model



The parameters of the Zipf-Mandelbrot model



The finite Zipf-Mandelbrot model

Evert (2004)

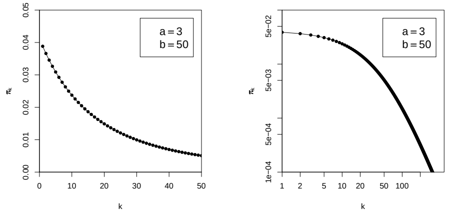
- Zipf-Mandelbrot population model characterizes an *infinite* type population: there is no upper bound on i, and the type probabilities π_i can become arbitrarily small
- $\pi = 10^{-6}$ (once every million words), $\pi = 10^{-9}$ (once every billion words), $\pi = 10^{-15}$ (once on the entire Internet), $\pi = 10^{-100}$ (once in the universe?)
- The **finite Zipf-Mandelbrot** model stops after first S types
- Population diversity S becomes a parameter of the model → the finite Zipf-Mandelbrot model has 3 parameters

Abbreviations:

- **ZM** for Zipf-Mandelbrot model
- **fZM** for finite Zipf-Mandelbrot model

Sampling from a population model

Assume we believe that the population we are interested in can be described by a Zipf-Mandelbrot model:



Use computer simulation to generate random samples:

- Draw N tokens from the population such that in each step, type w_i has probability π_i to be picked
- This allows us to make predictions for samples (= corpora) of arbitrary size N

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...
#3:	2	11	105	21	11	17	17	1	16	...
#4:	44	3	110	34	223	2	25	20	28	...
#5:	24	81	54	11	8	61	1	31	35	...
#6:	3	65	9	165	5	42	16	20	7	...
#7:	10	21	11	60	164	54	18	16	203	...
#8:	11	7	147	5	24	19	15	85	37	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Samples: type frequency list & spectrum

rank r	f_r	type i
1	37	6
2	36	1
3	33	3
4	31	7
5	31	10
6	30	5
7	28	12
8	27	2
9	24	4
10	24	16
11	23	8
12	22	14
⋮	⋮	⋮

m	V_m
1	83
2	22
3	20
4	12
5	10
6	5
7	5
8	3
9	3
10	3
⋮	⋮

sample #1

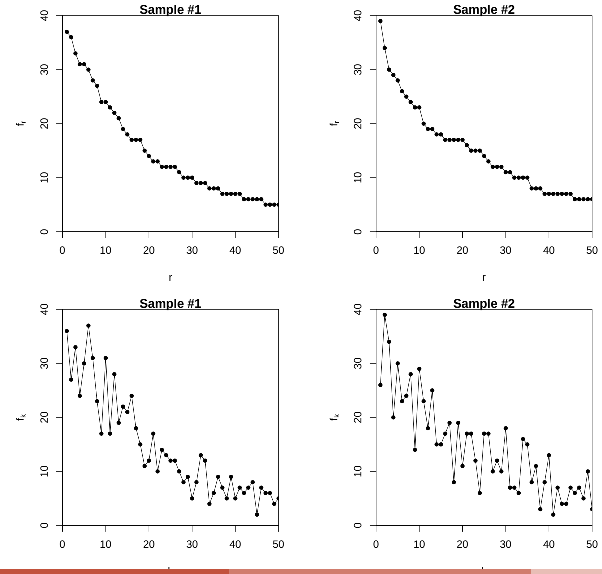
Samples: type frequency list & spectrum

rank r	f_r	type i
1	39	2
2	34	3
3	30	5
4	29	10
5	28	8
6	26	1
7	25	13
8	24	7
9	23	6
10	23	11
11	20	4
12	19	17
⋮	⋮	⋮

m	V_m
1	76
2	27
3	17
4	10
5	6
6	5
7	7
8	3
10	4
11	2
⋮	⋮

sample #2

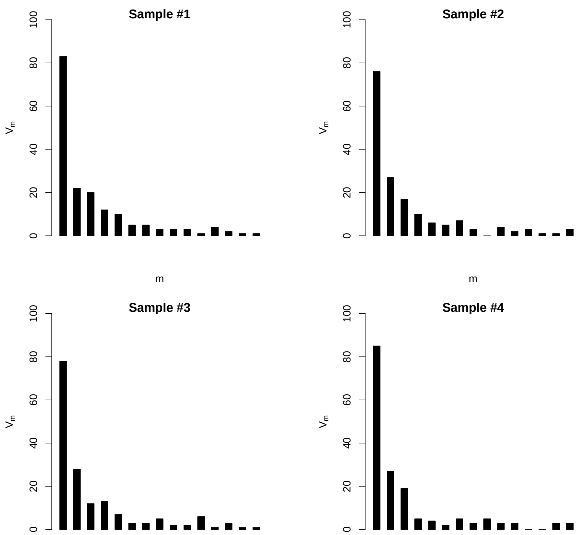
Random variation in type-frequency lists



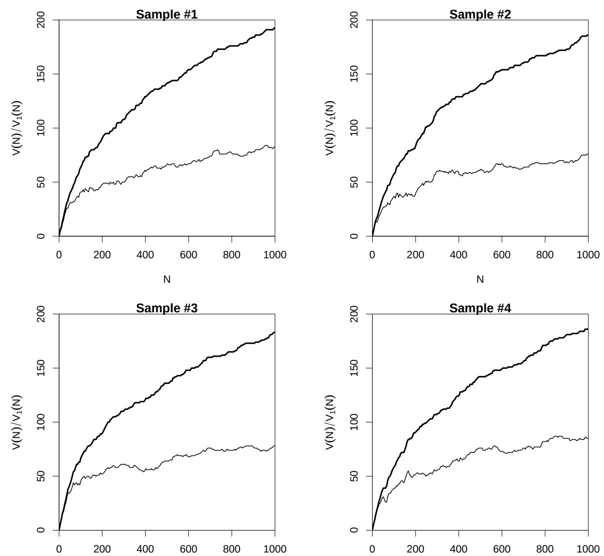
$r \leftrightarrow f_r$

$i \leftrightarrow f_i$

Random variation: frequency spectrum



Random variation: vocabulary growth curve



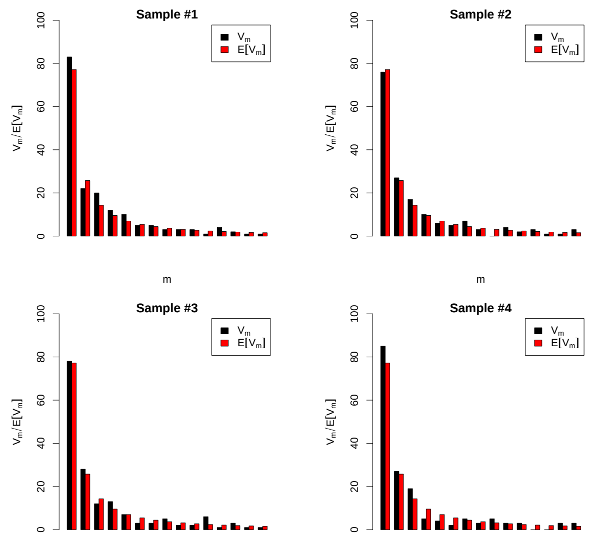
Expected values

- There is no reason why we should choose a particular sample to compare to the real data or make a prediction – each one is equally likely or unlikely
- Take the average over a large number of samples, called **expected value** or **expectation** in statistics

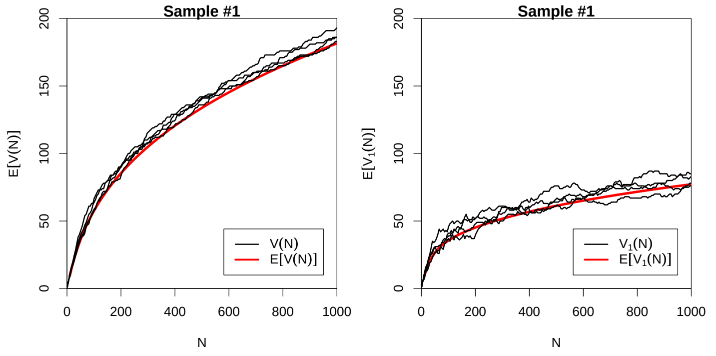
- Notation: $\mathrm{E}[V(N)]$ and $\mathrm{E}[V_m(N)]$
  - indicates that we are referring to expected values for a sample of size N
  - rather than to the specific values V and V_m observed in a particular sample or a real-world data set

- Expected values can be calculated efficiently *without* generating thousands of random samples

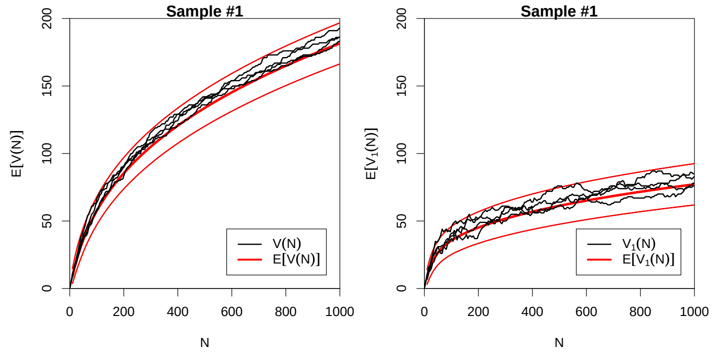
The expected frequency spectrum



The expected vocabulary growth curve



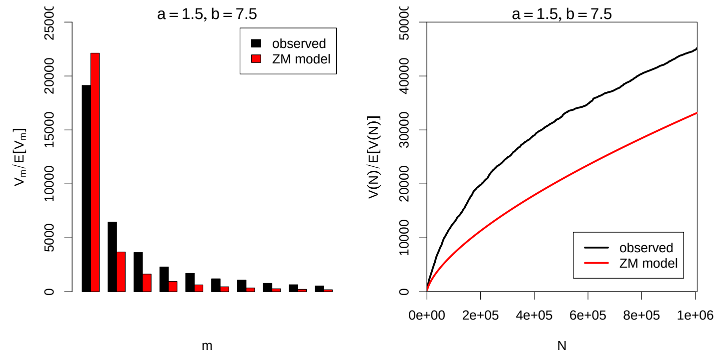
Prediction intervals for the expected VGC



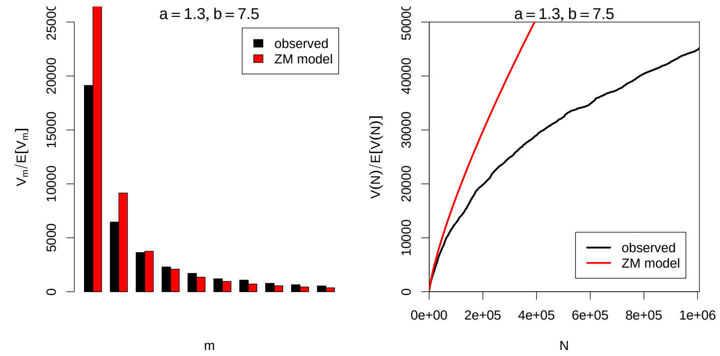
"Confidence intervals" indicate predicted sampling distribution:

☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines

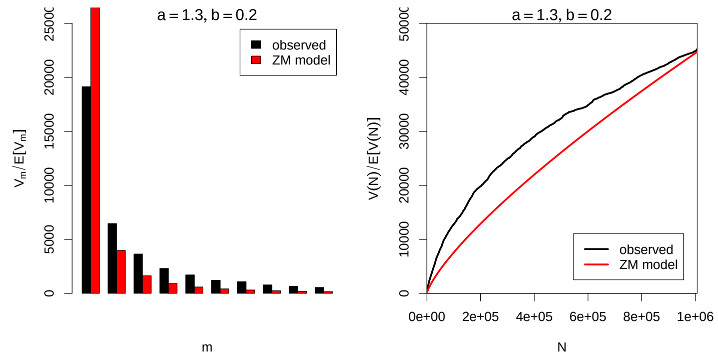
Parameter estimation by trial & error



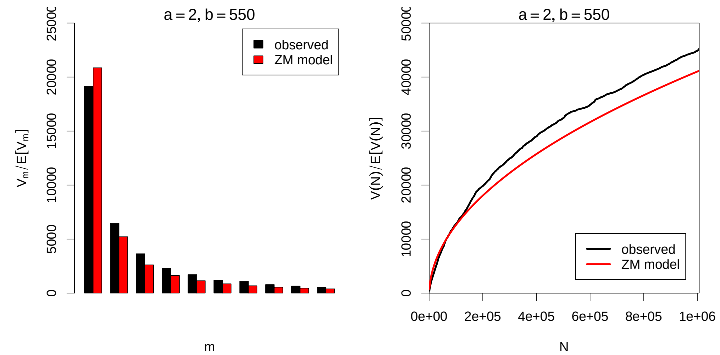
Parameter estimation by trial & error



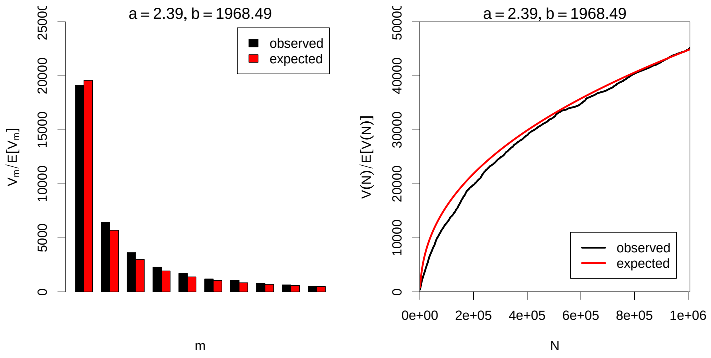
Parameter estimation by trial & error



Parameter estimation by trial & error



Automatic parameter estimation



- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure: $a = 2.39$ and $b = 1968$

Outline

The sampling model

- Draw random sample of N tokens from LNRE population
- Sufficient statistic: set of type frequencies $\{f_i\}$
  - because tokens of random sample have no ordering
- Joint **multinomial** distribution of $\{f_i\}$:

$$\Pr(\{f_i = k_i\} \mid N) = \frac{N!}{k_1! \cdots k_S!} \pi_1^{k_1} \cdots \pi_S^{k_S}$$

- **Approximation:** do not condition on fixed sample size N
  - N is now the average (expected) sample size
- Random variables f_i have **independent Poisson** distributions:

$$\Pr(f_i = k_i) = e^{-N\pi_i} \frac{(N\pi_i)^{k_i}}{k_i!}$$

Frequency spectrum

- Key problem: we cannot determine f_i in observed sample
  - becasue we don't know which type w_i is
  - recall that population ranking $f_i \neq$ Zipf ranking f_r
- Use spectrum $\{V_m\}$ and sample size V as statistics
  - contains all information we have about observed sample
- Can be expressed in terms of indicator variables

$$I_{[f_i=m]} = \begin{cases} 1 & f_i = m \\ 0 & \text{otherwise} \end{cases}$$

$$V_m = \sum_{i=1}^{S} I_{[f_i=m]}$$

$$V = \sum_{i=1}^{S} I_{[f_i>0]} = \sum_{i=1}^{S} (1 - I_{[f_i=0]})$$

The expected spectrum

- It is easy to compute expected values for the frequency spectrum (and variances because the f_i are independent)

$$\mathrm{E}[I_{[f_i=m]}] = \Pr(f_i = m) = e^{-N\pi_i}\frac{(N\pi_i)^m}{m!}$$

$$\mathrm{E}[V_m] = \sum_{i=1}^{S}\mathrm{E}[I_{[f_i=m]}] = \sum_{i=1}^{S} e^{-N\pi_i}\frac{(N\pi_i)^m}{m!}$$

$$\mathrm{E}[V] = \sum_{i=1}^{S}\mathrm{E}[1 - I_{[f_i=0]}] = \sum_{i=1}^{S}(1 - e^{-N\pi_i})$$

- NB: V_m and V are not independent because they are derived from the same random variables f_i

Sampling distribution of V_m and V

- Joint sampling distribution of $\{V_m\}$ and V is complicated
- **Approximation:** V and $\{V_m\}$ asymptotically follow a **multivariate normal** distribution
  - motivated by the multivariate central limit theorem: sum of many independent variables $I_{[f_i=m]}$
- Usually limited to first spectrum elements, e.g. $V_1, \dots, V_{15}$
  - approximation of discrete V_m by continuous distribution suitable only if $\mathrm{E}[V_m]$ is sufficiently large
- Parameters of multivariate normal: $\boldsymbol{\mu} = (\mathrm{E}[V], \mathrm{E}[V_1], \mathrm{E}[V_2], \dots)$ and $\boldsymbol{\Sigma}$ = covariance matrix

$$\Pr\big((V, V_1, \dots, V_k) = \mathbf{v}\big) \sim \frac{e^{-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{v}-\boldsymbol{\mu})}}{\sqrt{(2\pi)^{k+1}\det\boldsymbol{\Sigma}}}$$

Type density function

- Discrete sums of probabilities in $\mathrm{E}[V]$, $\mathrm{E}[V_m]$, ldots are inconvenient and computationally expensive
- **Approximation:** continuous **type density function** $g(\pi)$

$$|\{w_i \mid a \le \pi_i \le b\}| = \int_a^b g(\pi)\,d\pi$$

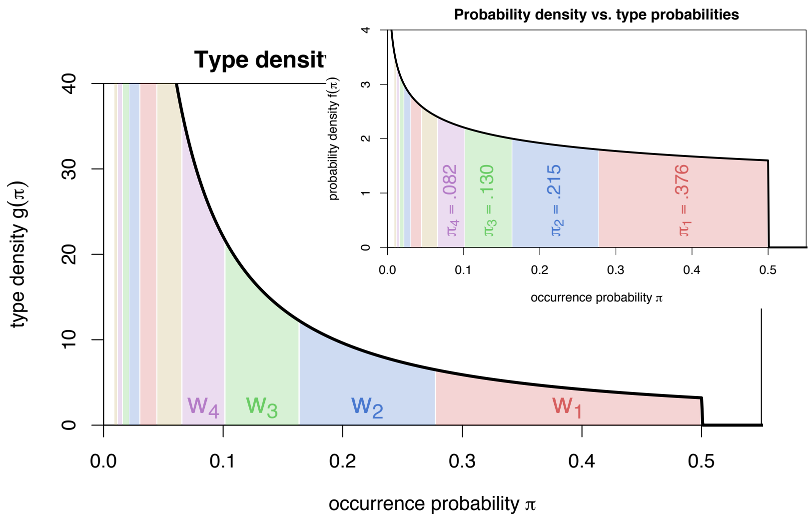
$$\sum\{\pi_i \mid a \le \pi_i \le b\} = \int_a^b \pi g(\pi)\,d\pi$$

- Normalization constraint:

$$\int_0^\infty \pi g(\pi)\,d\pi = 1$$

- Good approximation for low-probability types, but probability mass of $w_1, w_2, \dots$ "smeared out" over range

Type density function



ZM and fZM as LNRE models

- Discrete Zipf-Mandelbrot population

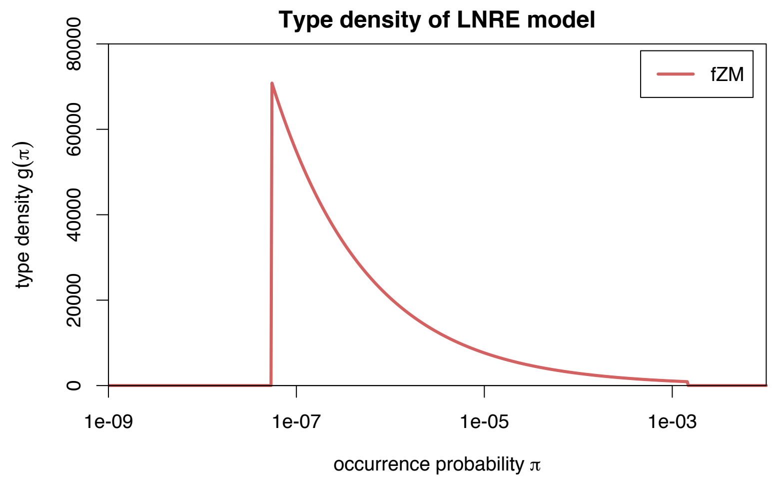
$$\pi_i := \frac{C}{(i+b)^a} \quad \text{for } i = 1, \ldots, S$$

- Corresponding type density function (Evert 2004)

$$g(\pi) = \begin{cases} C \cdot \pi^{-\alpha-1} & A \leq \pi \leq B \\ 0 & \text{otherwise} \end{cases}$$

with parameters
  - $\alpha = 1/a$ $(0 < \alpha < 1)$
  - $B = b \cdot \alpha/(1-\alpha)$
  - $0 \leq A < B$ determines S (ZM with $S = \infty$ for $A = 0$)
  - ☞ C is a normalization factor, not a parameter

ZM and fZM as LNRE models



Expectations as integrals

- Expected values can now be expressed as integrals over $g(\pi)$

$$\mathrm{E}[V_m] = \int_0^\infty \frac{(N\pi)^m}{m!} e^{-N\pi} g(\pi)\, d\pi$$

$$\mathrm{E}[V] = \int_0^\infty (1 - e^{-N\pi}) g(\pi)\, d\pi$$

- Reduce to simple closed form for ZM (approximation)

$$\mathrm{E}[V_m] = \frac{C}{m!} \cdot N^\alpha \cdot \Gamma(m - \alpha)$$

$$\mathrm{E}[V] = C \cdot N^\alpha \cdot \frac{\Gamma(1-\alpha)}{\alpha}$$

- fZM and exact solution for ZM with incompl. Gamma function

Parameter estimation from training corpus

- For ZM, $\alpha = \frac{\mathrm{E}[V_1]}{\mathrm{E}[V]} \approx \frac{V_1}{V}$ can be estimated directly, but prone to overfitting
- General parameter fitting by **MLE**: maximize likelihood of observed spectrum $\mathbf{v}$

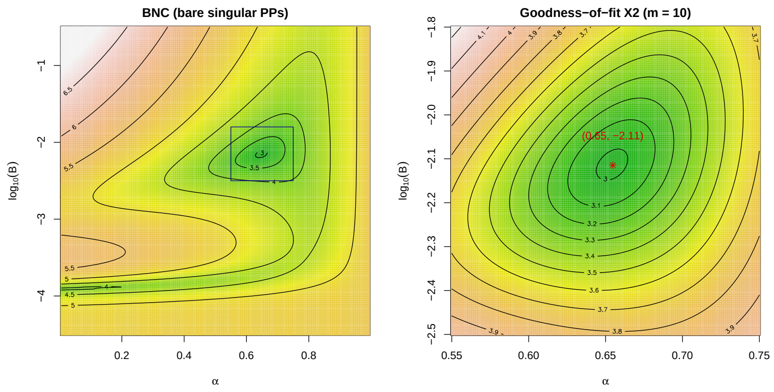
$$\max_{\alpha,A,B} \Pr\big((V, V1, \ldots, V_k) = \mathbf{v} \,\big|\, \alpha, A, B\big)$$

- Multivariate normal approximation:

$$\min_{\alpha,A,B} (\mathbf{v} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{v} - \boldsymbol{\mu})$$

- Minimization by gradient descent (BFGS, CG) or simplex search (Nelder-Mead)

Parameter estimation from training corpus



Goodness-of-fit

(Baayen 2001, Sec. 3.3)

- How well does the fitted model explain the observed data?
- For multivariate normal distribution:

$$X^2 = (\mathbf{V} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{V} - \boldsymbol{\mu}) \sim \chi^2_{k+1}$$

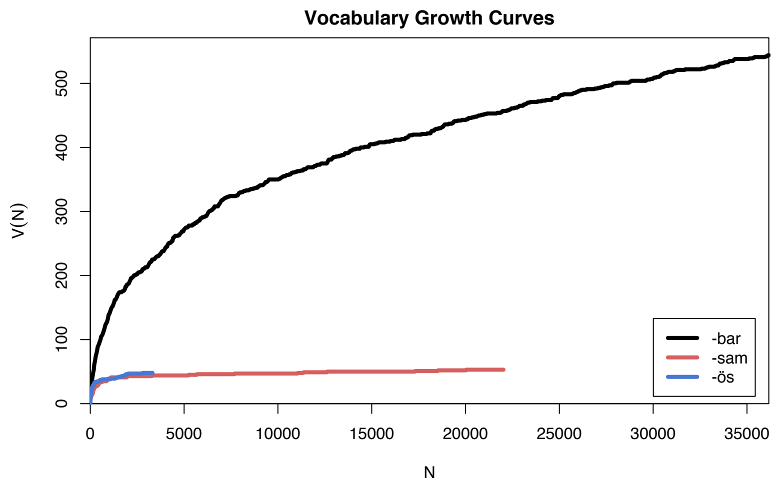
  where $\mathbf{V} = (V, V_1, \ldots, V_k)$
- ➥ Multivariate chi-squared test of **goodness-of-fit**
  - replace $\mathbf{V}$ by observed $\mathbf{v}$ ➜ test statistic x^2
  - must reduce df $= k + 1$ by number of estimated parameters
- NB: significant rejection of the LNRE model for $p < .05$

Coffee break!

Outline

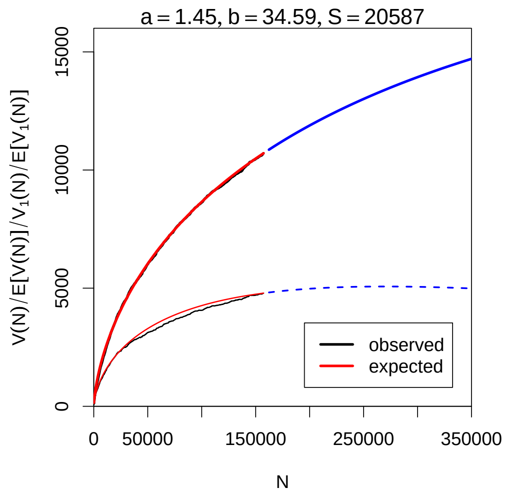
Measuring morphological productivity

example from Evert and Lüdeling (2001)



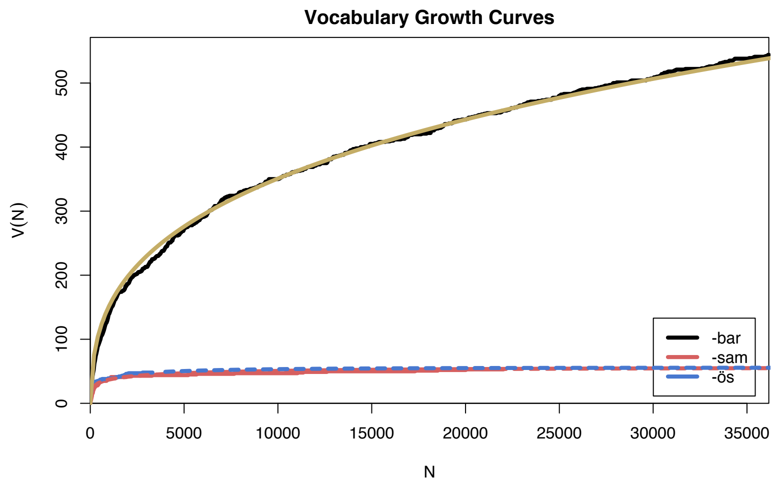
Measuring morphological productivity

example from Evert and Lüdeling (2001)



Measuring morphological productivity

example from Evert and Lüdeling (2001)



Quantitative measures of productivity

(Tweedie and Baayen 1998; Baayen 2001)

- Baayen's (1991) productivity index $\mathcal{P}$ (slope of vocabulary growth curve)

$$\mathcal{P} = \frac{V_1}{N}$$

- TTR = type-token ratio

$$\mathrm{TTR} = \frac{V}{N}$$

- Zipf-Mandelbrot slope

$$a$$

- Herdan's law (1964)

$$C = \frac{\log V}{\log N}$$

- Yule (1944) / Simpson (1949)

$$K = 10\,000 \cdot \frac{\sum_m m^2 V_m - N}{N^2}$$

- Guiraud (1954)

$$R = \frac{V}{\sqrt{N}}$$

- Sichel (1975)

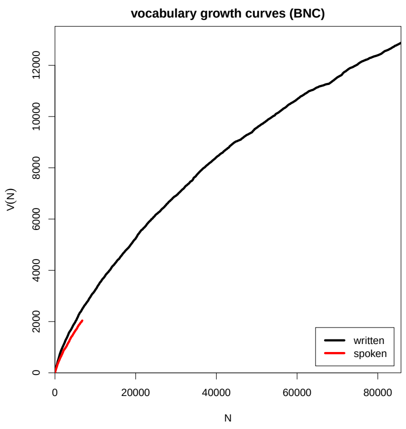
$$S = \frac{V_2}{V}$$

- Honoré (1979)

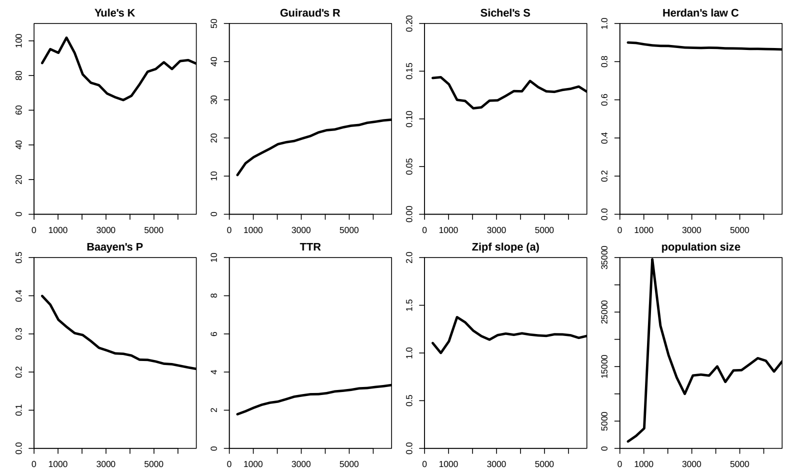
$$H = \frac{\log N}{1 - \frac{V_1}{V}}$$

Productivity measures for bare singulars in the BNC

	spoken	written
V	2,039	12,876
N	6,766	85,750
K	86.84	28.57
R	24.79	43.97
S	0.13	0.15
C	0.86	0.83
$\mathcal{P}$	0.21	0.08
TTR	0.301	0.150
a	1.18	1.27
pop. S	15,958	36,874



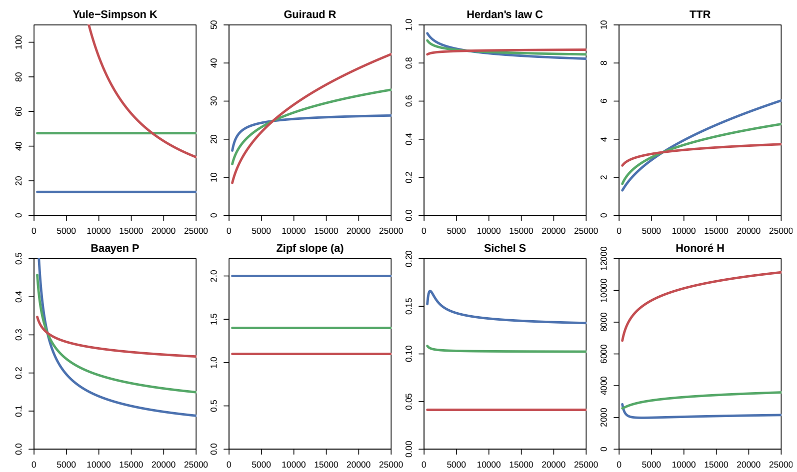
Are these "lexical constants" really constant?



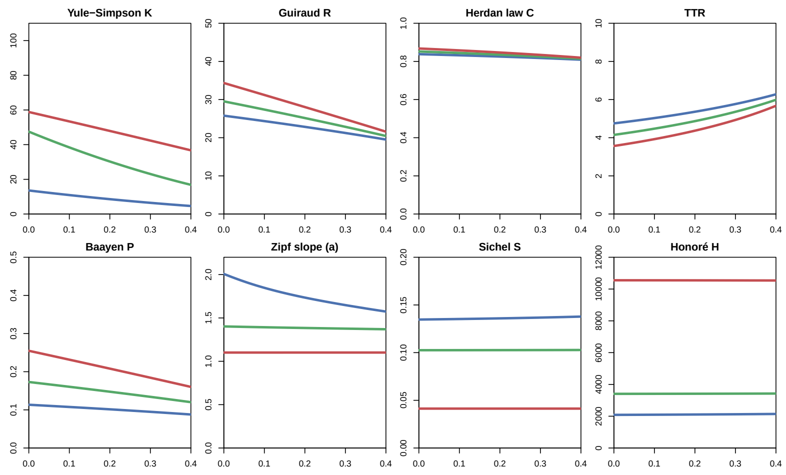
Simulation experiments based on LNRE models

- Systematic study of size dependence and other aspects of productivity measures based on samples from LNRE model
- LNRE model ➜ well-defined population
- Random sampling helps to assess variability of measures
- Expected values $\mathrm{E}[\mathcal{P}]$ etc. can often be computed directly (or approximated) ➜ computationally efficient
- ➥ LNRE models as tools for understanding productivity measures

Simulation: sample size

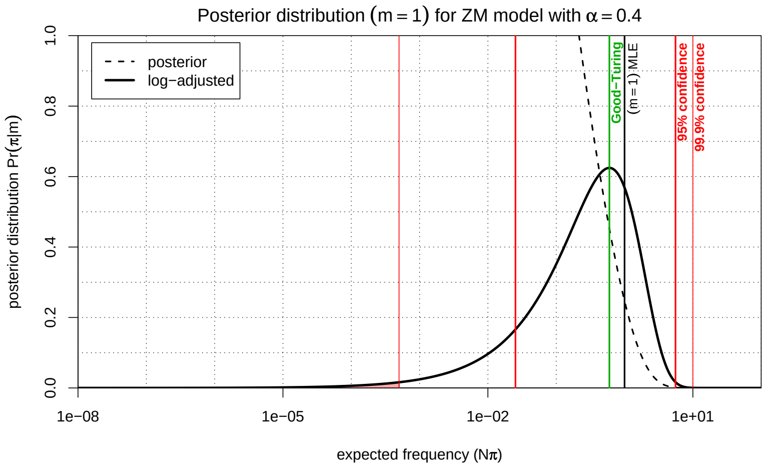


Simulation: frequent lexicalized types



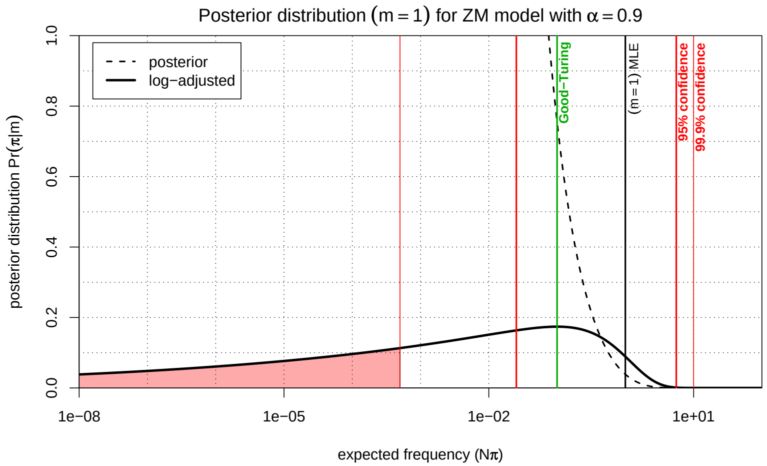
interactive demo

Posterior distribution



Part 2 Applications & examples (zipfR)

Posterior distribution



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Outline

How reliable are the fitted models?

Three potential issues:

1. Model assumptions ≠ population
   (e.g. distribution does not follow a Zipf-Mandelbrot law)
   - ☞ model cannot be adequate, regardless of parameter settings
2. Parameter estimation unsuccessful
   (i.e. suboptimal goodness-of-fit to training data)
   - ☞ optimization algorithm trapped in local minimum
   - ☞ can result in highly inaccurate model
3. Uncertainty due to sampling variation
   (i.e. training data differ from population distribution)
   - ☞ model fitted to training data, may not reflect true population
   - ☞ another training sample would have led to different parameters
   - ☞ especially critical for small samples ($N < 10{,}000$)

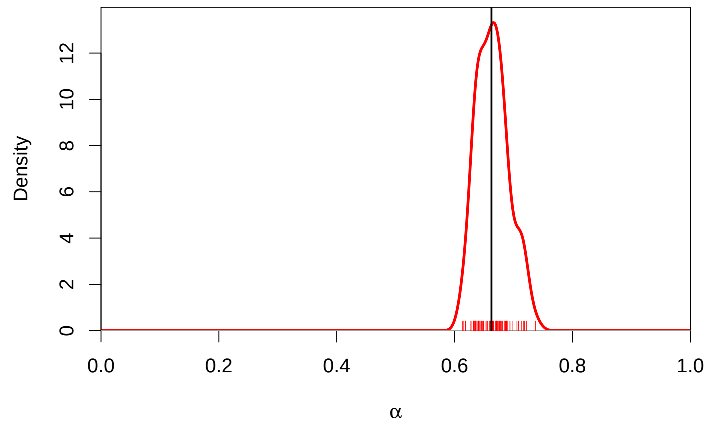
Bootstrapping

- An empirical approach to sampling variation:
  - take many random samples from the same population
  - estimate LNRE model from each sample
  - analyse distribution of model parameters, goodness-of-fit, etc. (mean, median, s.d., boxplot, histogram, ...)
  - problem: how to obtain the additional samples?
- Bootstrapping (Efron 1979)
  - resample from observed data *with replacement*
  - this approach is not suitable for type-token distributions (resamples underestimate vocabulary size V!)
- Parametric bootstrapping
  - use fitted model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known

Bootstrapping

parametric bootstrapping with 100 replicates

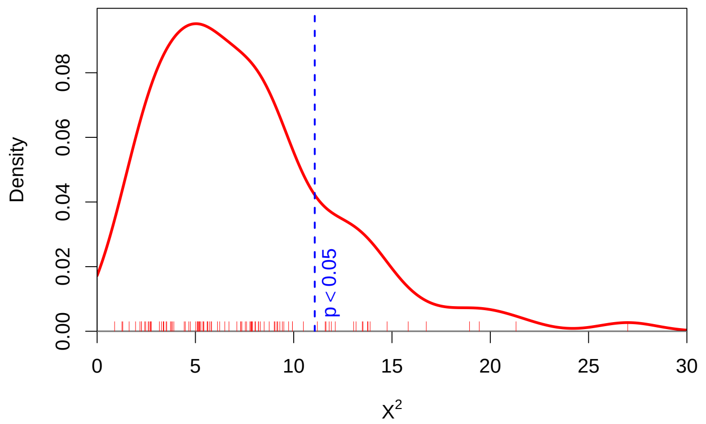
Zipfian slope $a = 1/\alpha$



Bootstrapping

parametric bootstrapping with 100 replicates

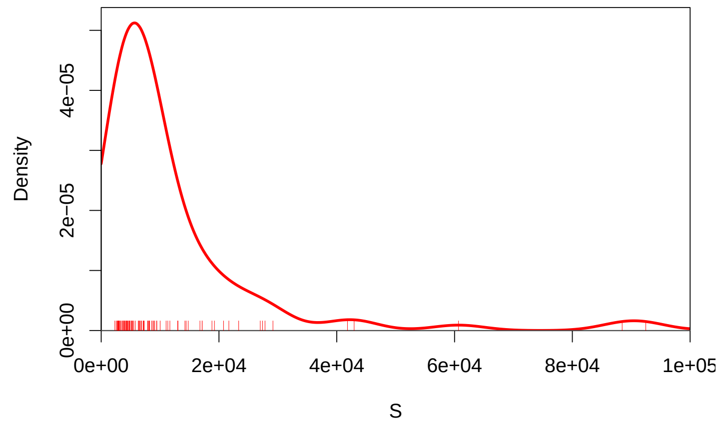
Goodness-of-fit statistic X^2 (model not plausible for $X^2 > 11$)



Bootstrapping

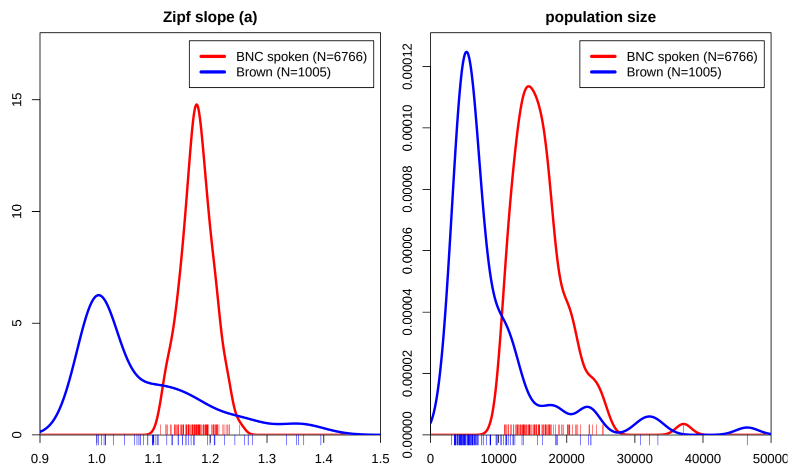
parametric bootstrapping with 100 replicates

Population diversity S



Sample size matters!

Brown corpus is too small for reliable LNRE parameter estimation (bare singulars)



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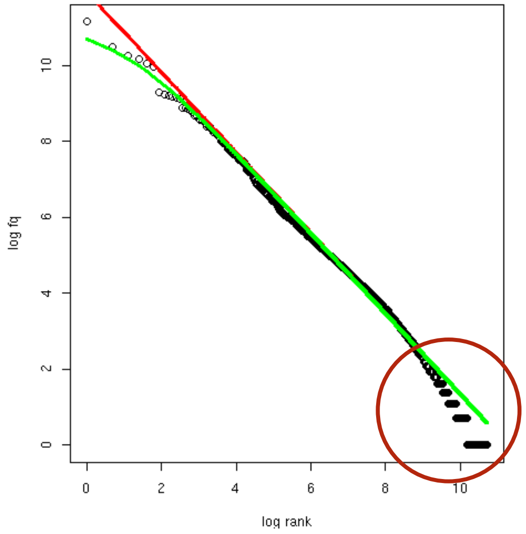
How well does Zipf's law hold?

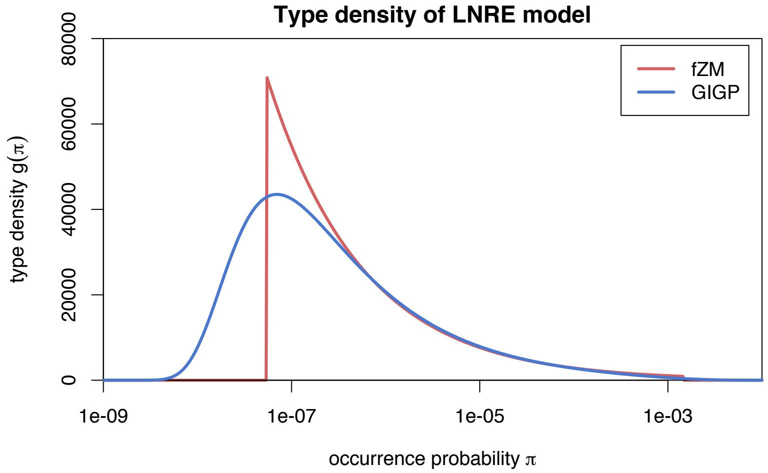


How well does Zipf's law hold?

- Z-M law seems to fit the first few thousand ranks very well, but then slope of empirical ranking becomes much steeper
  - similar patterns have been found in many different data sets
- Various modifications and extensions have been suggested (Sichel 1971; Kornai 1999; Montemurro 2001)
  - mathematics of corresponding LNRE models are often much more complex and numerically challenging
  - may not have closed form for $\mathrm{E}[V]$, $\mathrm{E}[V_m]$, or for the cumulative type distribution $G(\rho) = \int_\rho^\infty g(\pi)\, d\pi$
- E.g. Generalized Inverse Gauss-Poisson (GIGP; Sichel 1971)

$$g(\pi) = \frac{(2/bc)^{\gamma+1}}{K_{\gamma+1}(b)} \cdot \pi^{\gamma-1} \cdot e^{-\frac{\pi}{c} - \frac{b^2 c}{4\pi}}$$

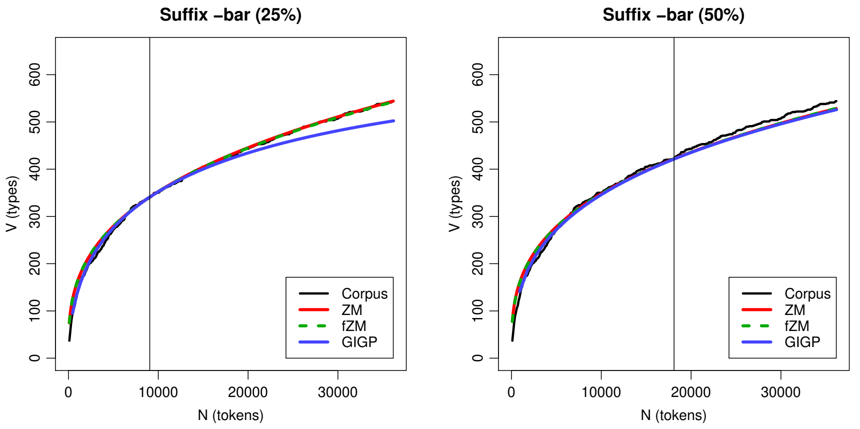
The GIGP model (Sichel 1971)



Outline

How accurate is LNRE-based extrapolation?

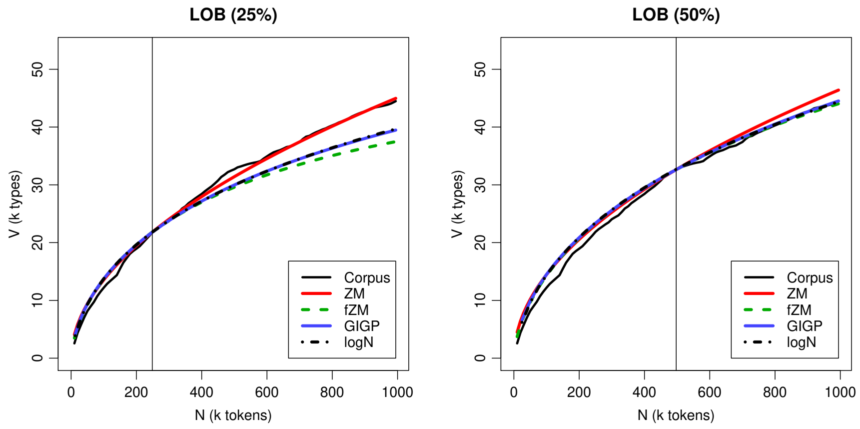
(Baroni and Evert 2005)



Part 2 Non-randomness

How accurate is LNRE-based extrapolation?

(Baroni and Evert 2005)



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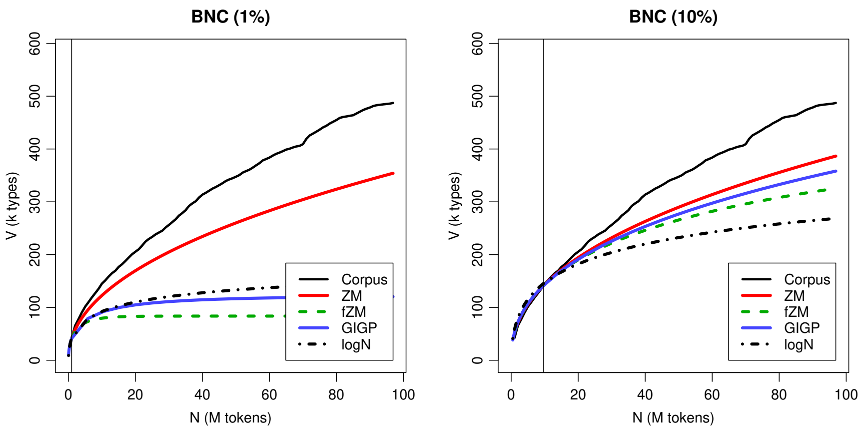
Part 2 Non-randomness

How accurate is LNRE-based extrapolation?

(Baroni and Evert 2005)



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Part 2 Non-randomness

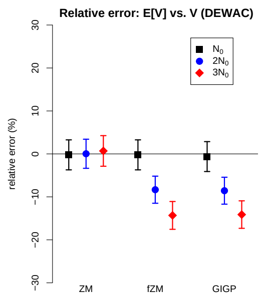
Reasons for poor extrapolation quality

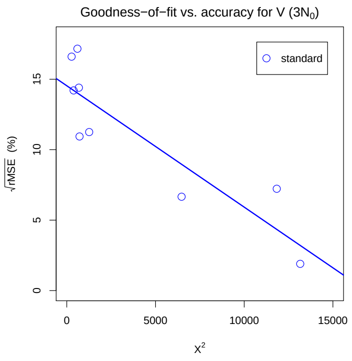
- Major problem: **non-randomness** of corpus data
  - LNRE modelling assumes that corpus is random sample
- Cause 1: **repetition** within texts
  - most corpora use entire text as unit of sampling
  - also referred to as "term clustering" or "burstiness"
  - well-known in computational linguistics (Church 2000)
- Cause 2: **non-homogeneous** corpus
  - cannot extrapolate from spoken BNC to written BNC
  - similar for different genres and domains
  - also within single text, e.g. beginning/end of novel

The ECHO correction

(Baroni and Evert 2007)

- Empirical study: quality of extrapolation $N_0 \to 4N_0$ starting from random samples of corpus texts

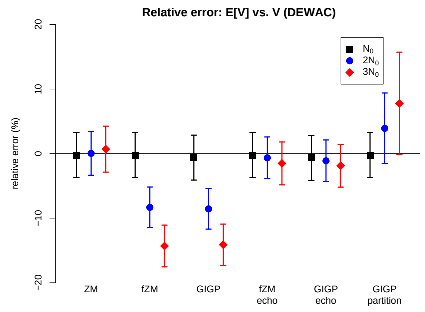


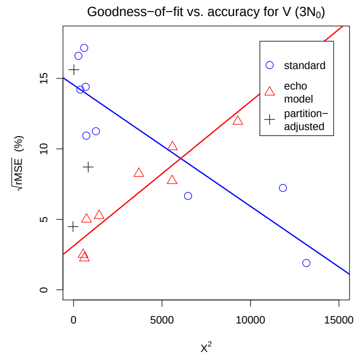


The ECHO correction

(Baroni and Evert 2007)

- ECHO correction: replace every repetition within same text by special type ECHO (= document frequencies)





Outline

Future plans for zipfR

- More efficient LNRE sampling & parametric bootstrapping
- Improve parameter estimation (minimization algorithm)
- Better computation accuracy by numerical integration
- Extended Zipf-Mandelbrot LNRE model: piecewise power law
- Development of robust and interpretable productivity measures, using LNRE simulations
- Computationally expensive modelling (MCMC) for accurate inference from small samples

Thank you!

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