What Every Computational Linguist Should Know About Type-Token Distributions and Zipf's Law

Tutorial 1, 7 May 2018

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http://zipfr.r-forge.r-project.org/lrec2018.html

Outline

Outline

Type-token statistics

- Type-token statistics different from most statistical inference
  - not about probability of a specific event
  - but about diversity of events and their probability distribution
- Relatively little work in statistical science
- Nor a major research topic in computational linguistics
  - very specialized, usually plays ancillary role in NLP
- But type-token statistics appear in wide range of applications
  - often crucial for sound analysis

➥ NLP community needs better awareness of statistical techniques, their limitations, and available software

Some research questions

- How many words did Shakespeare know?
- What is the coverage of my treebank grammar on big data?
- How many typos are there on the Internet?
- Is *-ness* more productive than *-ity* in English?
- Are there differences in the productivity of nominal compounds between academic writing and novels?
- Does Dickens use a more complex vocabulary than Rowling?
- Can a decline in lexical complexity predict Alzheimer's disease?
- How frequent is a hapax legomenon from the Brown corpus?
- What is appropriate smoothing for my n-gram model?
- Who wrote the Bixby letter, Lincoln or Hay?
- How many different species of . . . are there? (Brainerd 1982)

Some research questions

-
- coverage estimates
-
-
- productivity
- lexical complexity & stylometry
-
- prior & posterior distribution
-
- unexpected applications
-

Zipf's law (Zipf 1949)

A) Frequency distributions in natural language are highly skewed

B) Curious relationship between rank & frequency

word	r	f	$r \cdot f$
the	1.	142,776	142,776
and	2.	100,637	201,274
be	3.	94,181	282,543
of	4.	74,054	296,216

(Dickens)

C) Various explanations of Zipf's law

- principle of least effort (Zipf 1949)
- optimal coding system, MDL (Mandelbrot 1953, 1962)
- random sequences (Miller 1957; Li 1992; Cao *et al.* 2017)
- Markov processes ➜ n-gram models (Rouault 1978)

D) Language evolution: birth-death-process (Simon 1955)

☞ not the main topic today!

Outline

Tokens & types

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 15$: number of **tokens** = sample size
- $V = 7$: number of distinct **types** = **vocabulary size** (*recently*, *very*, *not*, *otherwise*, *much*, *merely*, *now*)

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type-frequency list

w	f_w
recently	1
very	5
not	3
otherwise	1
much	2
merely	2
now	1

Zipf ranking

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

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Zipf ranking

w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1

Zipf ranking

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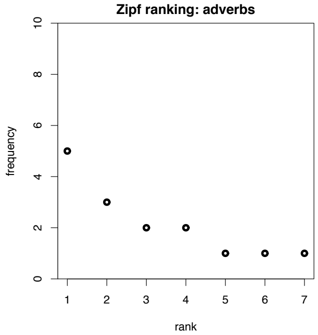

A realistic Zipf ranking: the Brown corpus

top frequencies			bottom frequencies		
r	***f***	**word**	**rank range**	***f***	**randomly selected examples**
1	69836	the	7731 – 8271	10	schedules, polynomials, bleak
2	36365	of	8272 – 8922	9	tolerance, shaved, hymn
3	28826	and	8923 – 9703	8	decreased, abolish, irresistible
4	26126	to	9704 – 10783	7	immunity, cruising, titan
5	23157	a	10784 – 11985	6	geographic, lauro, portrayed
6	21314	in	11986 – 13690	5	grigori, slashing, developer
7	10777	that	13691 – 15991	4	sheath, gaulle, ellipsoids
8	10182	is	15992 – 19627	3	mc, initials, abstracted
9	9968	was	19628 – 26085	2	thar, slackening, deluxe
10	9801	he	26086 – 45215	1	beck, encompasses, second-place

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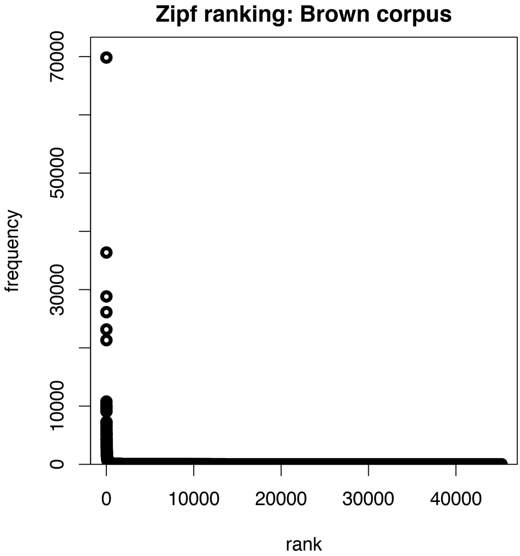

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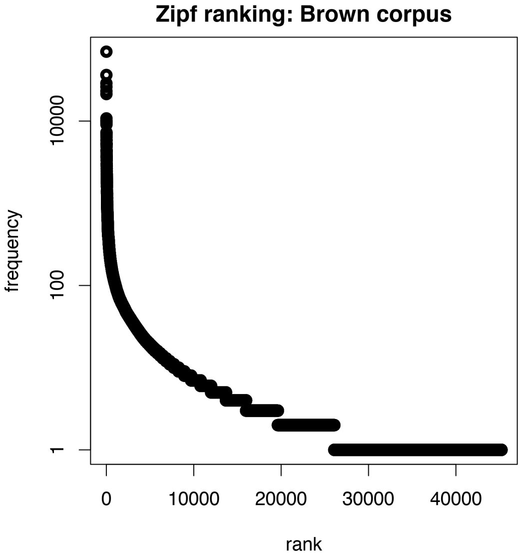

Frequency spectrum

- pool types with $f = 1$ (hapax legomena), types with $f = 2$ (dis legomena), ..., $f = m$, ...
- $V_1 = 3$: number of hapax legomena (*now, otherwise, recently*)
- $V_2 = 2$: number of dis legomena (*merely, much*)
- general definition: $V_m = |\{w \mid f_w = m\}|$

Zipf ranking

w	r	f_r
very	1	5
not	2	3
merely	3	2
much	4	2
now	5	1
otherwise	6	1
recently	7	1

frequency spectrum

m	V_m
1	3
2	2
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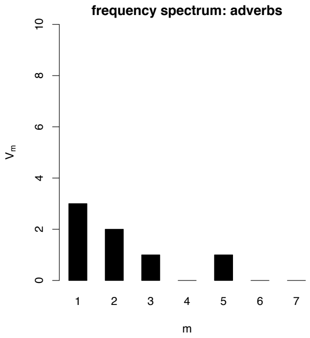

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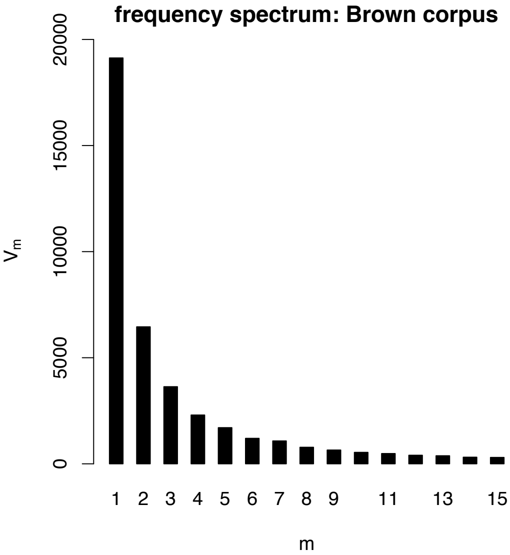

Vocabulary growth curve

our sample: *recently*, *very*, *not*, *otherwise*, *much*, *very*, *very*, *merely*, *not*, *now*, *very*, *much*, *merely*, *not*, *very*

- $N = 1$, $V(N) = 1$, $V_1(N) = 1$

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- $N = 3$, $V(N) = 3$, $V_1(N) = 3$
- $N = 7$, $V(N) = 5$, $V_1(N) = 4$

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- $N = 7,\ V(N) = 5,\ V_1(N) = 4$
- $N = 12,\ V(N) = 7,\ V_1(N) = 4$

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- $N = 15,\ V(N) = 7,\ V_1(N) = 3$

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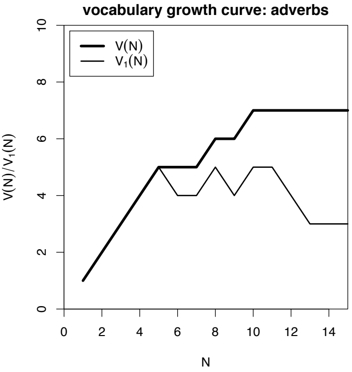
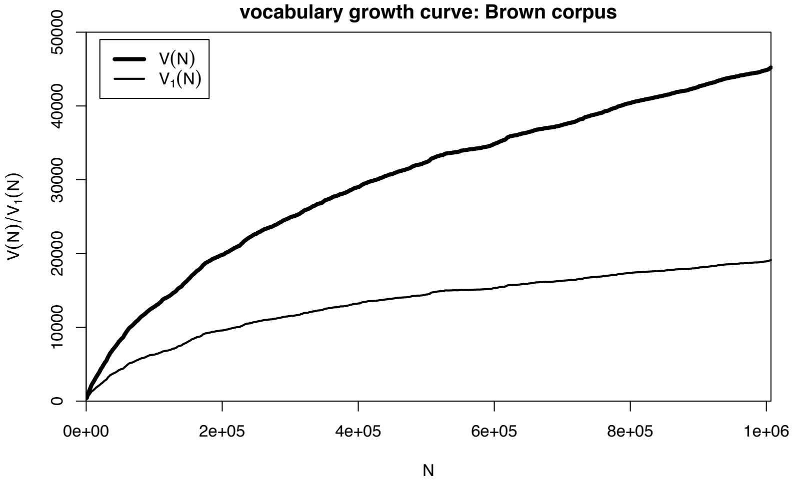
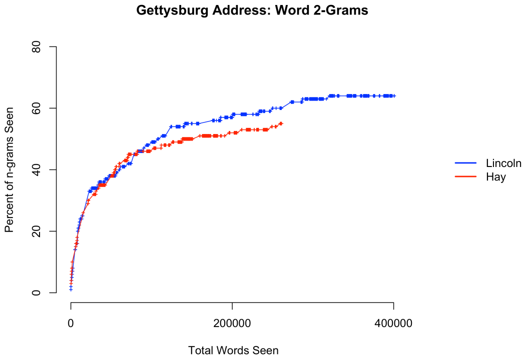


A realistic vocabulary growth curve: the Brown corpus



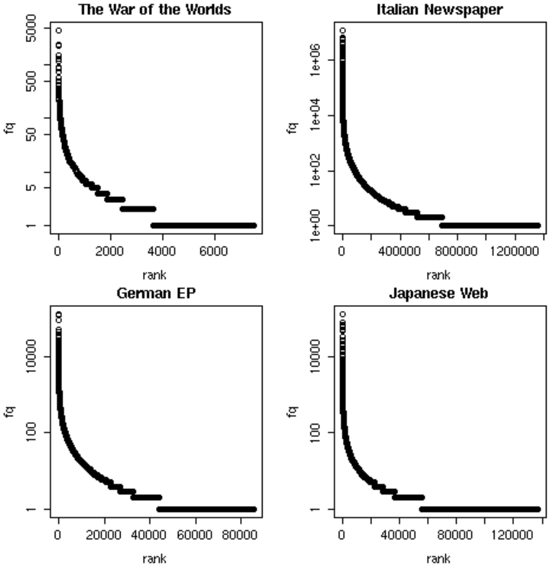
Vocabulary growth in authorship attribution

- Authorship attribution by n-gram tracing applied to the case of the Bixby letter (Grieve *et al.* submitted)
- Word or character n-grams in disputed text are compared against large "training" corpora from candidate authors



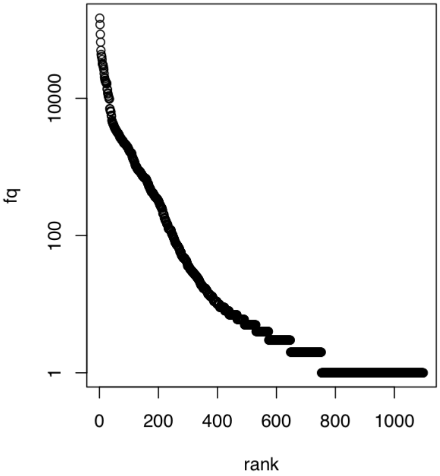
Observing Zipf's law

across languages and different linguistic units



Observing Zipf's law

The Italian prefix *ri-* in the *la Repubblica* corpus



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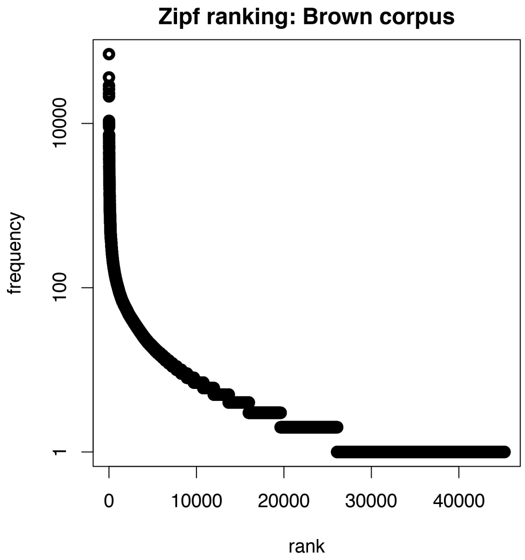

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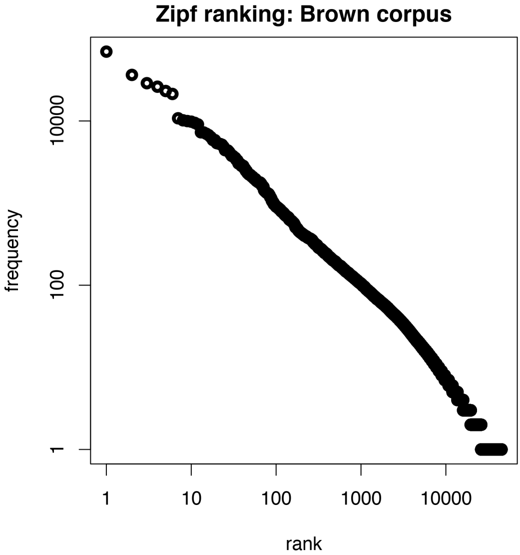

Observing Zipf's law

- Straight line in double-logarithmic space corresponds to **power law** for original variables
- This leads to Zipf's (1949; 1965) famous law:

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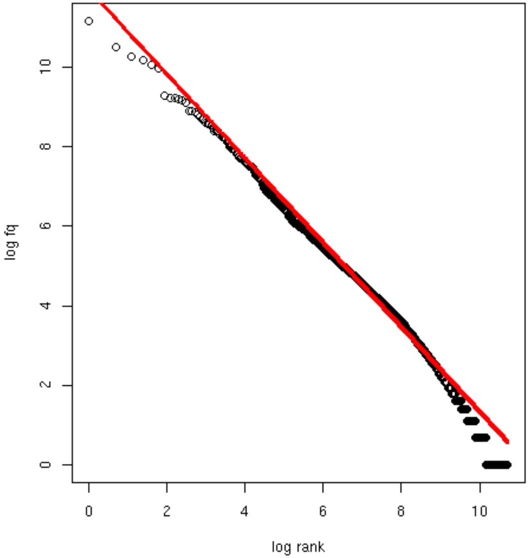
- Intuitive interpretation of a and C:
  - a is **slope** determining how fast log frequency decreases
  - $\log C$ is **intercept**, i.e. log frequency of most frequent word ($r = 1$ ➜ $\log r = 0$)

Observing Zipf's law

Least-squares fit = linear regression in log-space (Brown corpus)



Zipf-Mandelbrot law

Mandelbrot (1953, 1962)

- Mandelbrot's extra parameter:

$$f_r = \frac{C}{(r+b)^a}$$

- Zipf's law is special case with $b = 0$

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$$f_r = \frac{C}{(r+b)^a}$$

- Zipf's law is special case with $b = 0$
- Assuming $a = 1$, $C = 60{,}000$, $b = 1$:
  - For word with rank 1, Zipf's law predicts frequency of 60,000; Mandelbrot's variation predicts frequency of 30,000
  - For word with rank 1,000, Zipf's law predicts frequency of 60; Mandelbrot's variation predicts frequency of 59.94

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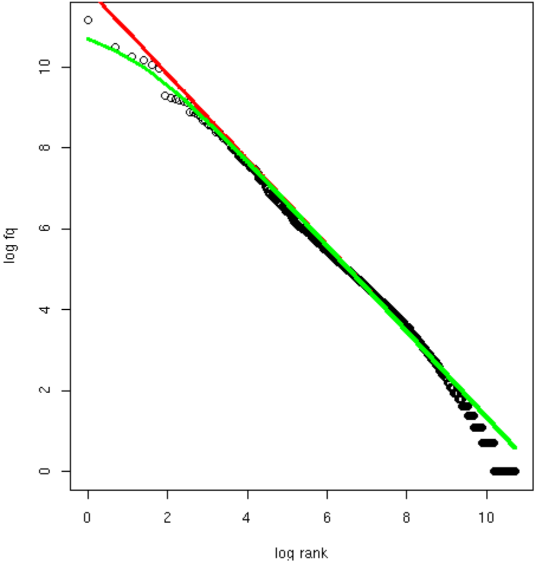
  - For word with rank 1,000, Zipf's law predicts frequency of 60; Mandelbrot's variation predicts frequency of 59.94
- Zipf-Mandelbrot law forms basis of statistical LNRE models
  - ZM law derived mathematically as limiting distribution of vocabulary generated by a character-level Markov process

Zipf-Mandelbrot law

Non-linear least-squares fit (Brown corpus)



Outline

zipfR
Evert and Baroni (2007)

- http://zipfR.R-Forge.R-Project.org/
- Conveniently available from CRAN repository
- Package vignette = gentle tutorial introduction

First steps with zipfR

- Set up a folder for this course, and make sure it is your working directory in R (preferably as an RStudio project)
- Install the most recent version of the zipfR package
- Package, handouts, code samples & data sets available from `http://zipfr.r-forge.r-project.org/lrec2018.html`

```
> library(zipfR)

> ?zipfR    # documentation entry point

> vignette("zipfr-tutorial")    # read the zipfR tutorial
```

Loading type-token data

- Most convenient input: sequence of tokens as text file in vertical format ("one token per line")
  - ☞ mapped to appropriate types: normalized word forms, word pairs, lemmatized, semantic class, n-gram of POS tags, ...
  - ☞ language data should always be in UTF-8 encoding!
  - ☞ large files can be compressed (`.gz`, `.bz2`, `.xz`)

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  - lowercased adverb tokens from Brown corpus (original order)
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```
> adv <- readLines("brown_adverbs.txt", encoding="UTF-8")

> head(adv, 30)  # mathematically, a ‘‘vector’’ of tokens
> length(adv)    # sample size = 52,037 tokens
```

Descriptive statistics: type-frequency list

```
> adv.tfl <- vec2tfl(adv)
> adv.tfl
    k    f type
1   1 4859  not
2   2 2084  n't
3   3 1464   so
4   4 1381 only
5   5 1374 then
6   6 1309  now
7   7 1134 even
8   8 1089   as
    ⋮    ⋮    ⋮
     N    V
 52037 1907

> N(adv.tfl)   # sample size
> V(adv.tfl)   # type count
```

Descriptive statistics: frequency spectrum

```
> adv.spc <- tfl2spc(adv.tfl)   # or directly with vec2spc
> adv.spc
    m  Vm
1   1 762
2   2 260
3   3 144
4   4  99
5   5  69
6   6  50
7   7  40
8   8  34
    ⋮   ⋮
     N    V
 52037 1907

> N(adv.spc)   # sample size
> V(adv.spc)   # type count
```

Descriptive statistics: vocabulary growth

- VGC lists vocabulary size $V(N)$ at different sample sizes N
- Optionally also spectrum elements $V_m(N)$ up to `m.max`

```
> adv.vgc <- vec2vgc(adv, m.max=2)
```

- Visualize descriptive statistics with `plot` method

```
> plot(adv.tfl)                  # Zipf ranking
> plot(adv.tfl, log="xy")        # logarithmic scale recommended

> plot(adv.spc)                  # barplot of frequency spectrum

> plot(adv.vgc, add.m = 1:2)     # vocabulary growth curve
```

Further example data sets

`?Brown`	words from Brown corpus
`?BrownSubsets`	various subsets
`?Dickens`	words from novels by Charles Dickens
`?ItaPref`	Italian word-formation prefixes
`?TigerNP`	NP and PP patterns from German Tiger treebank
`?Baayen2001`	frequency spectra from Baayen (2001)
`?EvertLuedeling2001`	German word-formation affixes (manually corrected data from Evert and Lüdeling 2001)

Practice:

- Explore these data sets with descriptive statistics
- Try different plot options (from help pages `?plot.tfl`, `?plot.spc`, `?plot.vgc`)

Outline

Motivation

- Interested in productivity of affix, vocabulary of author, . . . ; not in a particular text or sample
  - ☞ statistical inference from sample to population
- Discrete frequency counts are difficult to capture with generalizations such as Zipf's law
  - Zipf's law predicts many impossible types with $1 < f_r < 2$
  - ☞ population does not suffer from such quantization effects

LNRE models

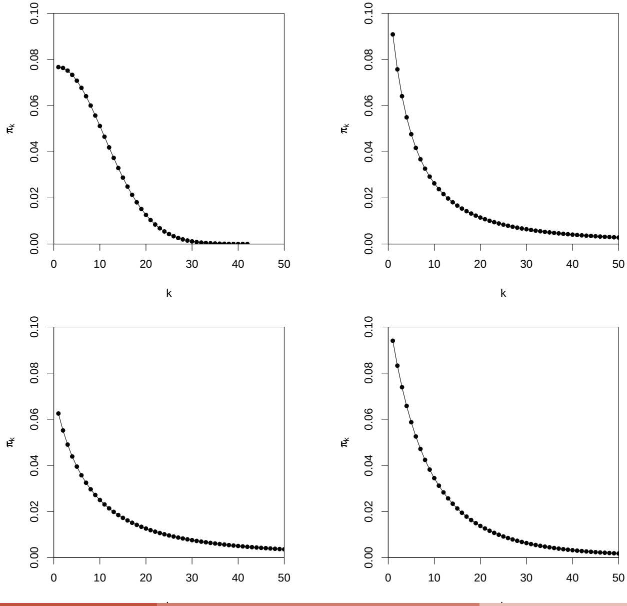
- This tutorial introduces the state-of-the-art LNRE approach proposed by Baayen (2001)
  - LNRE = Large Number of Rare Events
- LNRE uses various approximations and simplifications to obtain a tractable and elegant model
- Of course, we could also estimate the precise discrete distributions using MCMC simulations, but …
  1. LNRE model usually minor component of complex procedure
  2. often applied to very large samples ($N > 1$ M tokens)

The LNRE population

- Population: set of S types w_i with occurrence **probabilities** π_i
- S = **population diversity** can be finite or infinite ($S = \infty$)
- Not interested in specific types ➜ arrange by decreasing probability: $\pi_1 \geq \pi_2 \geq \pi_3 \geq \cdots$
  - ☞ impossible to determine probabilities of all individual types
- Normalization: $\pi_1 + \pi_2 + \ldots + \pi_S = 1$

- Need **parametric** statistical **model** to describe full population (esp. for $S = \infty$), i.e. a function $i \mapsto \pi_i$
  - type probabilities π_i cannot be estimated reliably from a sample, but parameters of this function can
  - NB: population index $i \neq$ Zipf rank r

Examples of population models



The Zipf-Mandelbrot law as a population model

What is the right family of models for lexical frequency distributions?

- We have already seen that the Zipf-Mandelbrot law captures the distribution of observed frequencies very well

The Zipf-Mandelbrot law as a population model

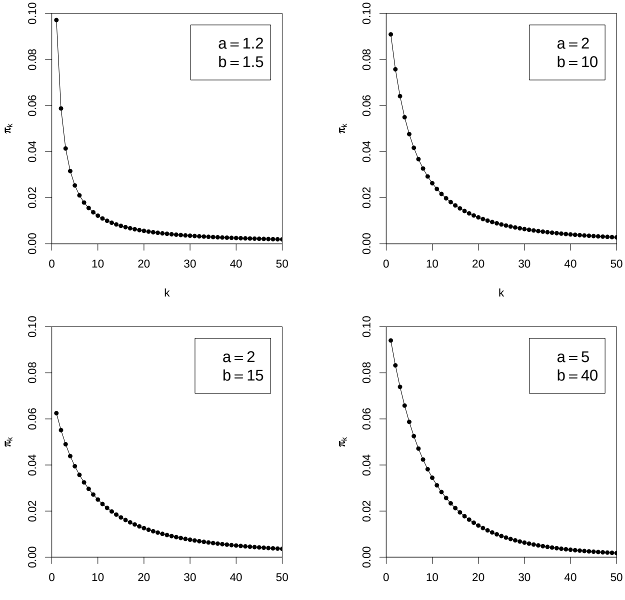
What is the right family of models for lexical frequency distributions?

- We have already seen that the Zipf-Mandelbrot law captures the distribution of observed frequencies very well
- Re-phrase the law for type probabilities:

$$\pi_i := \frac{C}{(i+b)^a}$$

- Two free parameters: $a > 1$ and $b \geq 0$
- C is not a parameter but a normalization constant, needed to ensure that $\sum_i \pi_i = 1$
- This is the **Zipf-Mandelbrot** population model

The parameters of the Zipf-Mandelbrot model



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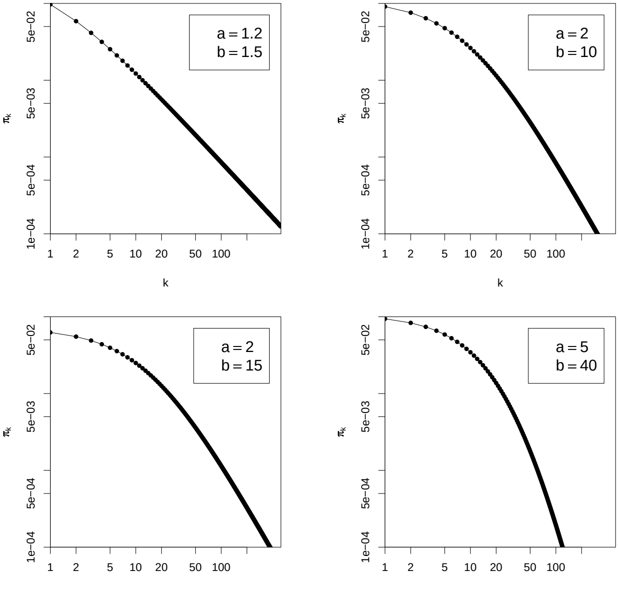

The finite Zipf-Mandelbrot model

Evert (2004)

- Zipf-Mandelbrot population model characterizes an *infinite* type population: there is no upper bound on i, and the type probabilities π_i can become arbitrarily small
- $\pi = 10^{-6}$ (once every million words), $\pi = 10^{-9}$ (once every billion words), $\pi = 10^{-15}$ (once on the entire Internet), $\pi = 10^{-100}$ (once in the universe?)

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- Population diversity S becomes a parameter of the model $\rightarrow$ the finite Zipf-Mandelbrot model has 3 parameters

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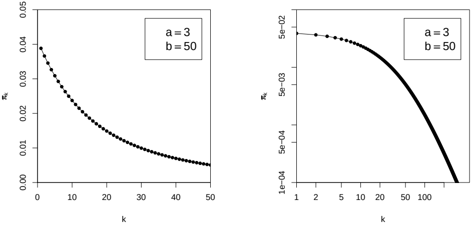
Abbreviations:

- **ZM** for Zipf-Mandelbrot model
- **fZM** for finite Zipf-Mandelbrot model

Sampling from a population model

Assume we believe that the population we are interested in can be described by a Zipf-Mandelbrot model:



Use computer simulation to generate random samples:

- Draw N tokens from the population such that in each step, type w_i has probability π_i to be picked
- This allows us to make predictions for samples (= corpora) of arbitrary size N

Sampling from a population model

#1: 1 42 34 23 108 18 48 18 1 ...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	...
	time	order	room	school	town	course	area	course	time	...
#2:	286	28	23	36	3	4	7	4	8	...
#3:	2	11	105	21	11	17	17	1	16	...

Sampling from a population model

#1:	1	42	34	23	108	18	48	18	1	. . .
	time	order	room	school	town	course	area	course	time	. . .
#2:	286	28	23	36	3	4	7	4	8	. . .
#3:	2	11	105	21	11	17	17	1	16	. . .
#4:	44	3	110	34	223	2	25	20	28	. . .
#5:	24	81	54	11	8	61	1	31	35	. . .
#6:	3	65	9	165	5	42	16	20	7	. . .
#7:	10	21	11	60	164	54	18	16	203	. . .
#8:	11	7	147	5	24	19	15	85	37	. . .
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	

Samples: type frequency list & spectrum

rank r	f_r	type i
1	37	6
2	36	1
3	33	3
4	31	7
5	31	10
6	30	5
7	28	12
8	27	2
9	24	4
10	24	16
11	23	8
12	22	14
⋮	⋮	⋮

m	V_m
1	83
2	22
3	20
4	12
5	10
6	5
7	5
8	3
9	3
10	3
⋮	⋮

sample #1

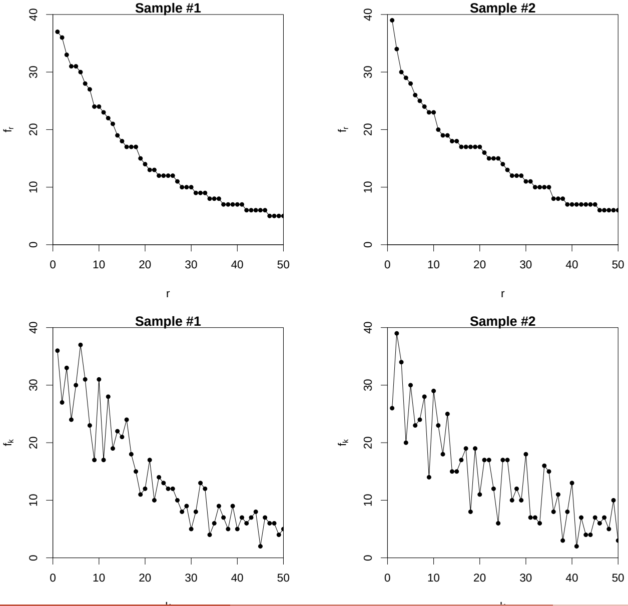
Samples: type frequency list & spectrum

rank r	f_r	type i
1	39	2
2	34	3
3	30	5
4	29	10
5	28	8
6	26	1
7	25	13
8	24	7
9	23	6
10	23	11
11	20	4
12	19	17
⋮	⋮	⋮

m	V_m
1	76
2	27
3	17
4	10
5	6
6	5
7	7
8	3
10	4
11	2
⋮	⋮

sample #2

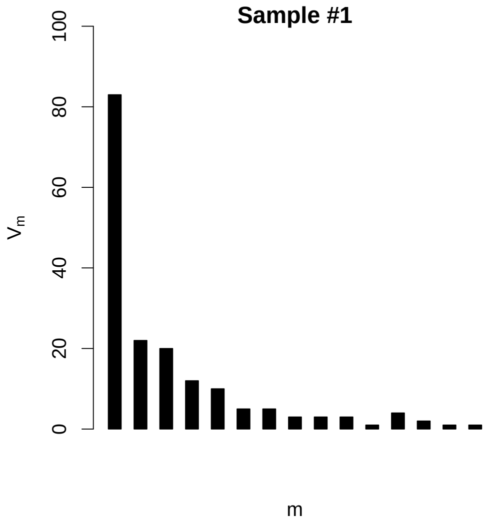
Random variation in type-frequency lists



$r \leftrightarrow f_r$

$i \leftrightarrow f_i$

Random variation: frequency spectrum



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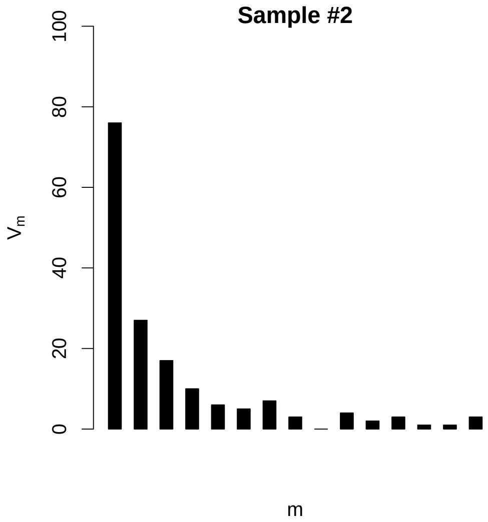

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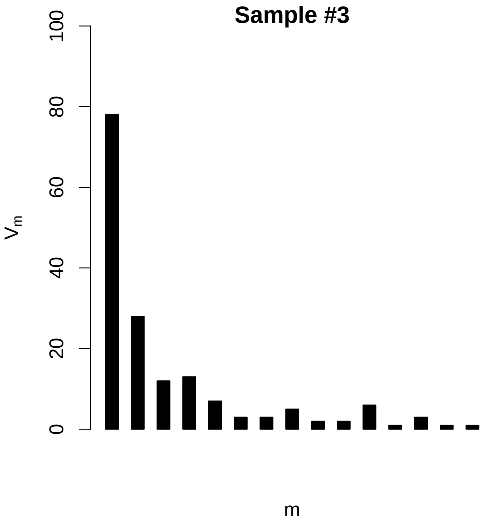
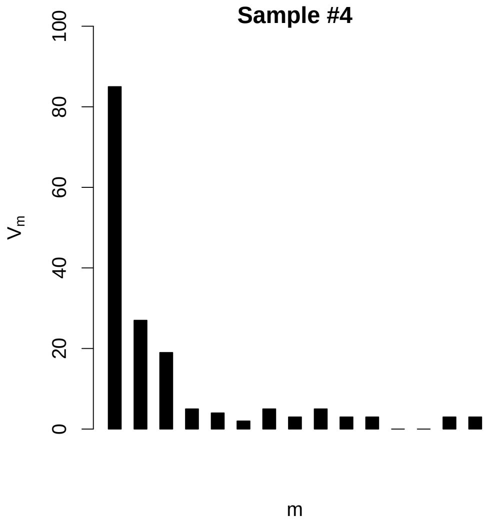
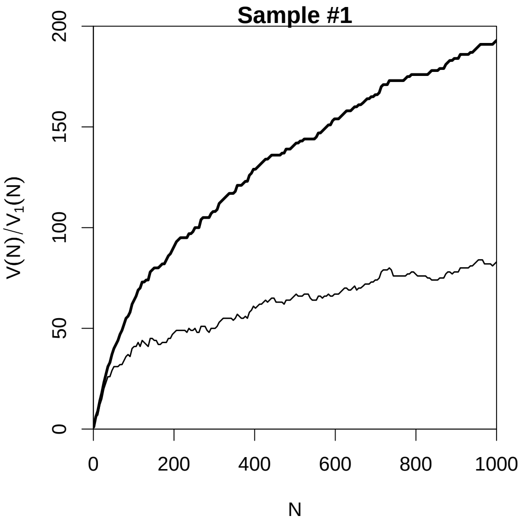


Random variation: frequency spectrum



Random variation: vocabulary growth curve



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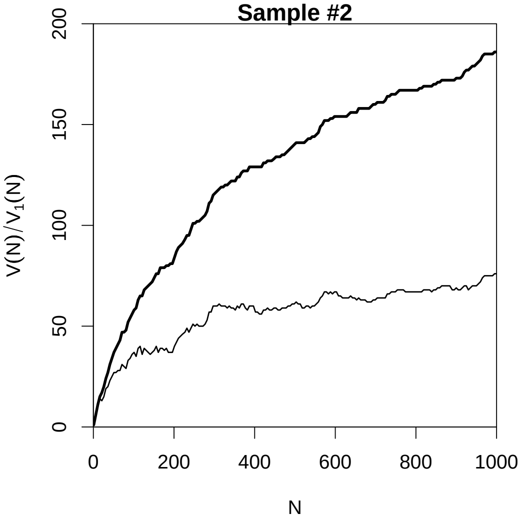

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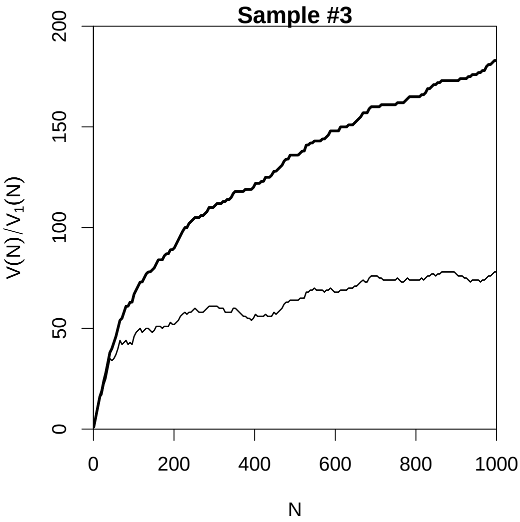

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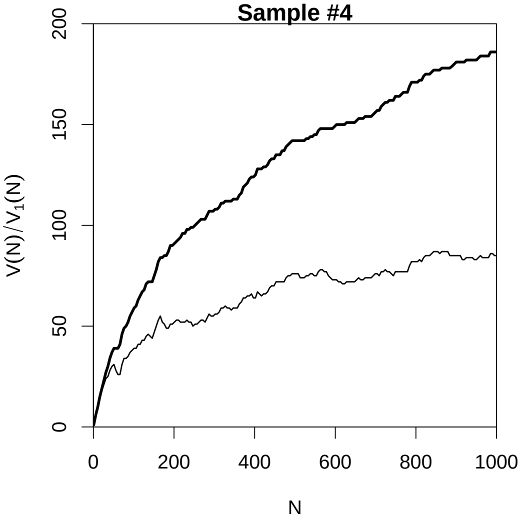


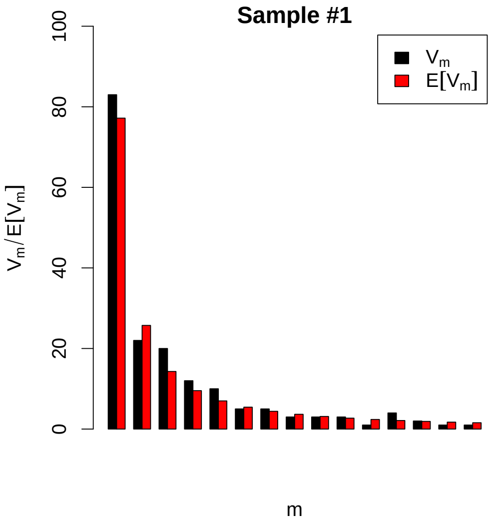
Expected values

- There is no reason why we should choose a particular sample to compare to the real data or make a prediction – each one is equally likely or unlikely
- Take the average over a large number of samples, called **expected value** or **expectation** in statistics

- Notation: $\mathrm{E}[V(N)]$ and $\mathrm{E}[V_m(N)]$
  - indicates that we are referring to expected values for a sample of size N
  - rather than to the specific values V and V_m observed in a particular sample or a real-world data set

- Expected values can be calculated efficiently *without* generating thousands of random samples

The expected frequency spectrum



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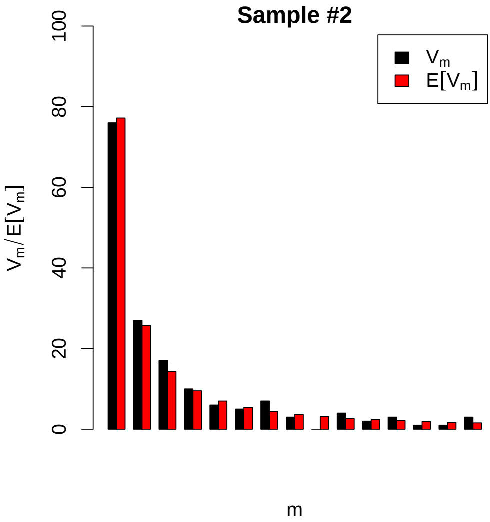

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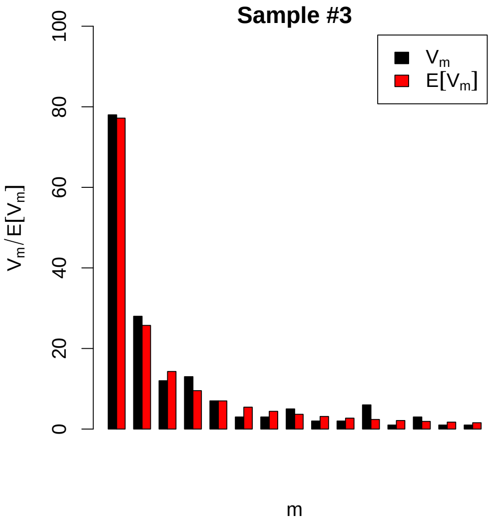

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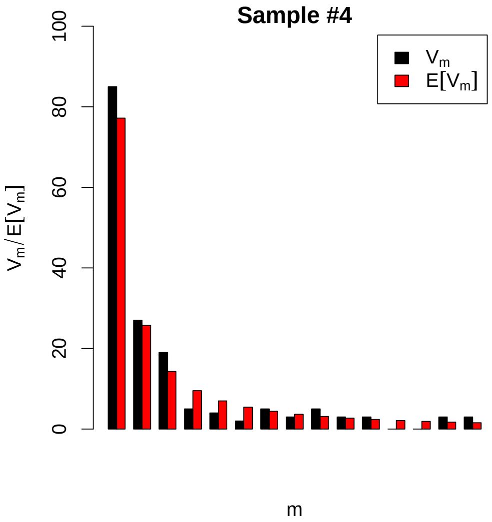
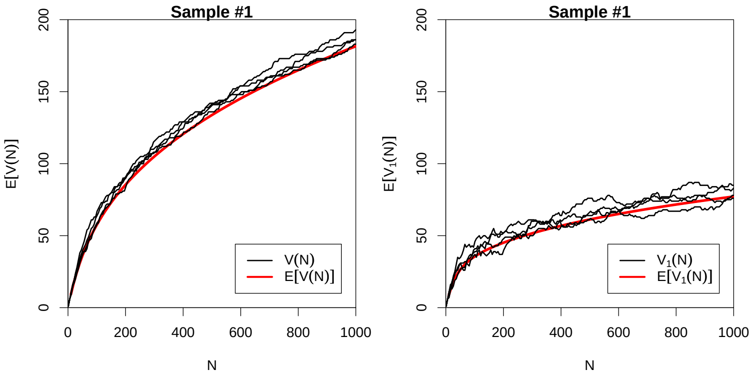
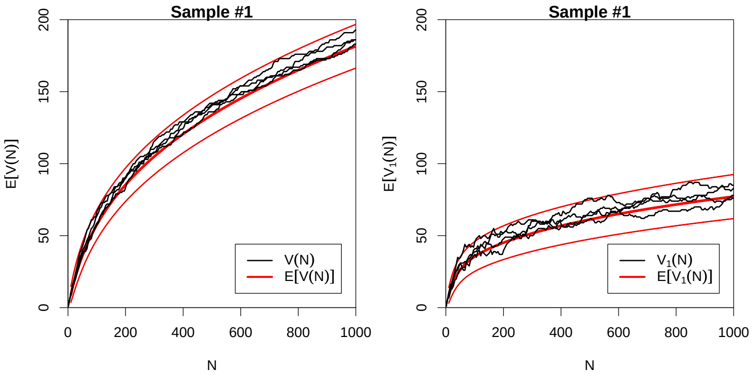


The expected vocabulary growth curve



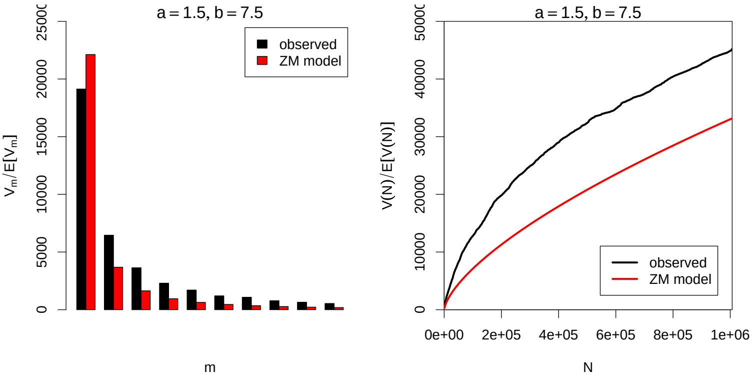
Prediction intervals for the expected VGC



"Confidence intervals" indicate predicted sampling distribution:

☞ for 95% of samples generated by the LNRE model, VGC will fall within the range delimited by the thin red lines

Parameter estimation by trial & error



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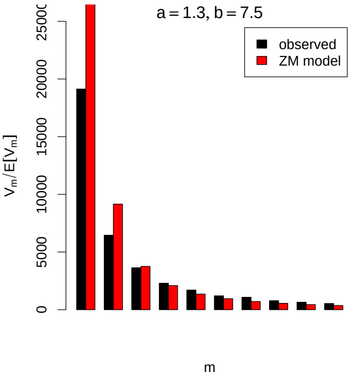

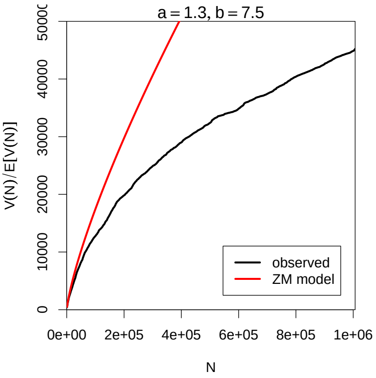

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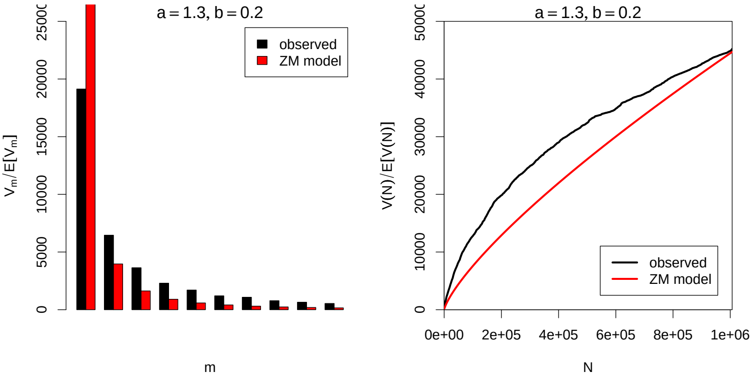

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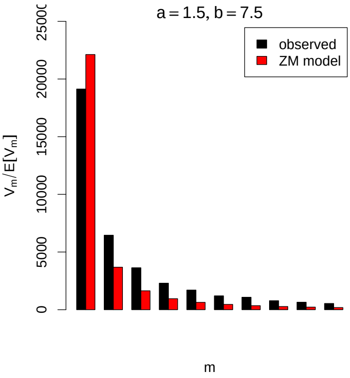

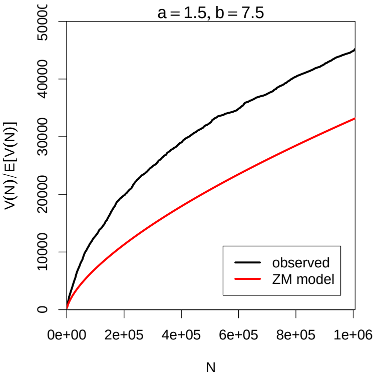

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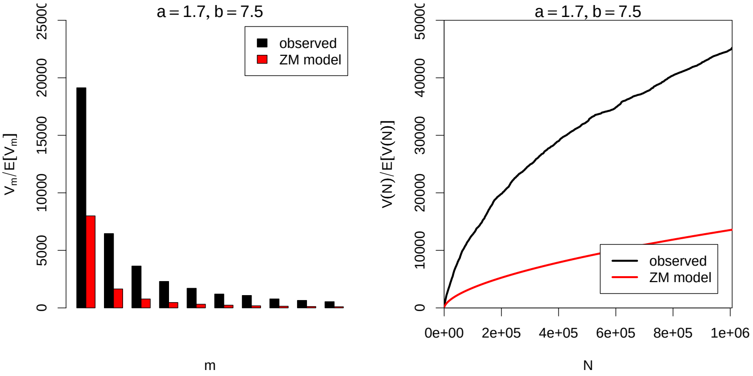

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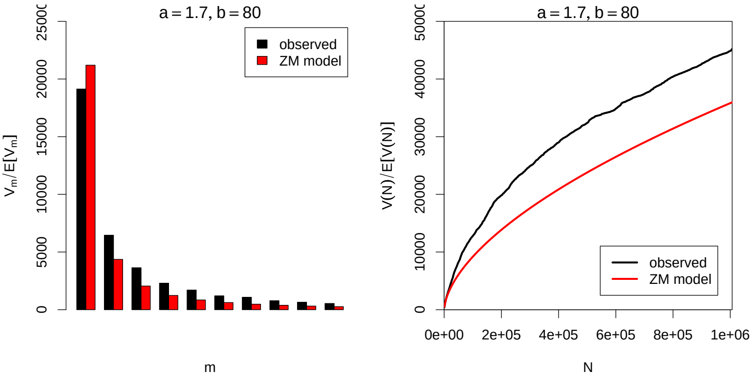

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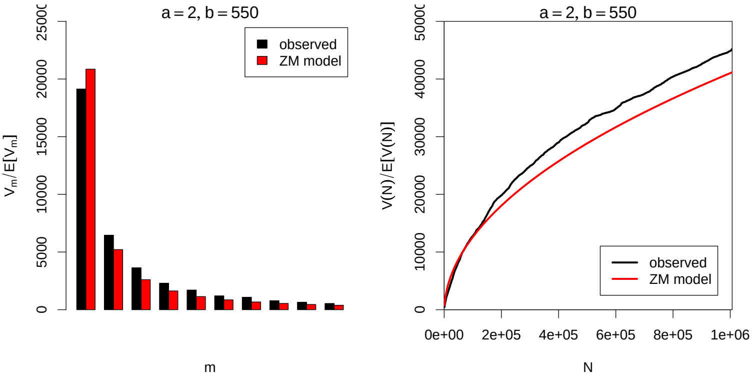
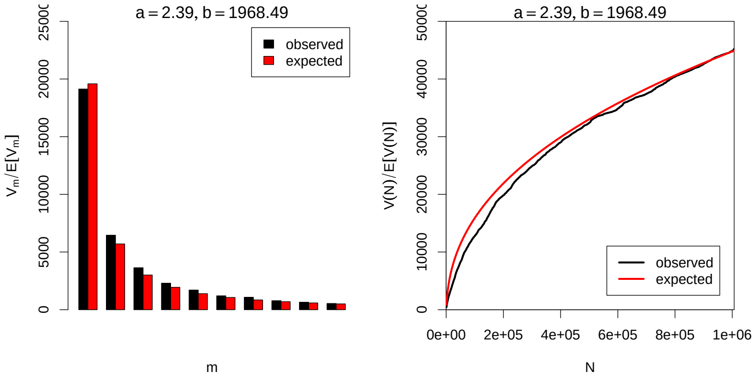


Automatic parameter estimation



- By trial & error we found $a = 2.0$ and $b = 550$
- Automatic estimation procedure: $a = 2.39$ and $b = 1968$

Outline

The sampling model

- Draw random sample of N tokens from LNRE population
- Sufficient statistic: set of type frequencies $\{f_i\}$
  - because tokens of random sample have no ordering
- Joint **multinomial** distribution of $\{f_i\}$:

$$\Pr(\{f_i = k_i\} \mid N) = \frac{N!}{k_1! \cdots k_S!} \pi_1^{k_1} \cdots \pi_S^{k_S}$$

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- **Approximation:** do not condition on fixed sample size N
  - N is now the average (expected) sample size
- Random variables f_i have **independent Poisson** distributions:

$$\Pr(f_i = k_i) = e^{-N\pi_i} \frac{(N\pi_i)^{k_i}}{k_i!}$$

Frequency spectrum

- Key problem: we cannot determine f_i in observed sample
  - becasue we don't know which type w_i is
  - recall that population ranking $f_i \neq$ Zipf ranking f_r
- Use spectrum $\{V_m\}$ and sample size V as statistics
  - contains all information we have about observed sample

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$$V = \sum_{i=1}^{S} I_{[f_i>0]} = \sum_{i=1}^{S} \bigl(1 - I_{[f_i=0]}\bigr)$$

The expected spectrum

- It is easy to compute expected values for the frequency spectrum (and variances because the f_i are independent)

$$\mathrm{E}[I_{[f_i=m]}] = \Pr(f_i = m) = e^{-N\pi_i} \frac{(N\pi_i)^m}{m!}$$

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- NB: V_m and V are not independent because they are derived from the same random variables f_i

Sampling distribution of V_m and V

- Joint sampling distribution of $\{V_m\}$ and V is complicated
- **Approximation:** V and $\{V_m\}$ asymptotically follow a **multivariate normal** distribution
  - motivated by the multivariate central limit theorem: sum of many independent variables $I_{[f_i=m]}$
- Usually limited to first spectrum elements, e.g. $V_1, \ldots, V_{15}$
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  - approximation of discrete V_m by continuous distribution suitable only if $\mathrm{E}[V_m]$ is sufficiently large
- Parameters of multivariate normal: $\boldsymbol{\mu} = (\mathrm{E}[V], \mathrm{E}[V_1], \mathrm{E}[V_2], \ldots)$ and $\boldsymbol{\Sigma}$ = covariance matrix

$$\Pr\big((V, V_1, \ldots, V_k) = \mathbf{v}\big) \sim \frac{e^{-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{v}-\boldsymbol{\mu})}}{\sqrt{(2\pi)^{k+1} \det \boldsymbol{\Sigma}}}$$

Type density function

- Discrete sums of probabilities in $\mathrm{E}[V]$, $\mathrm{E}[V_m]$, ldots are inconvenient and computationally expensive
- **Approximation:** continuous **type density function** $g(\pi)$

$$|\{w_i \,|\, a \leq \pi_i \leq b\}| = \int_a^b g(\pi)\, d\pi$$

$$\sum\{\pi_i \,|\, a \leq \pi_i \leq b\} = \int_a^b \pi g(\pi)\, d\pi$$

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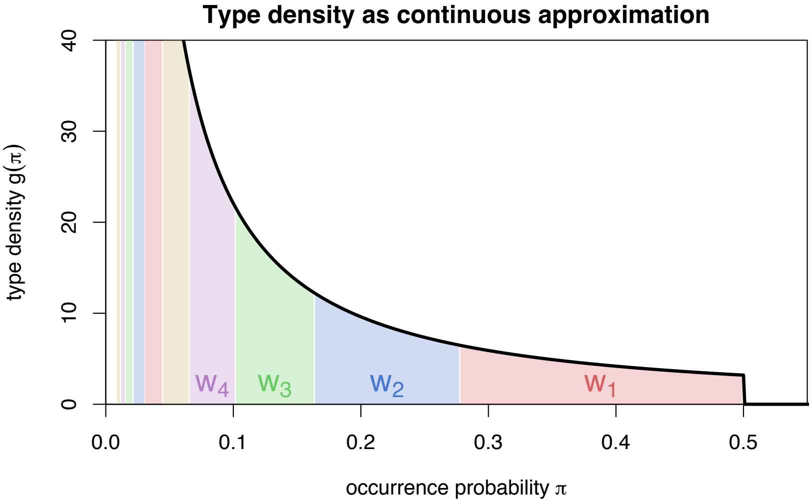
- Normalization constraint:

$$\int_0^\infty \pi g(\pi)\, d\pi = 1$$

- Good approximation for low-probability types, but probability mass of $w_1, w_2, \ldots$ "smeared out" over range

Type density function



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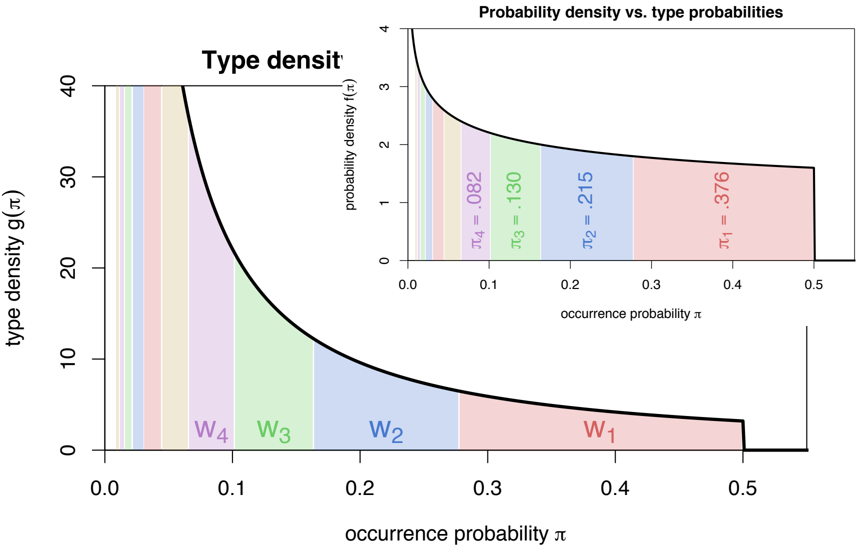

ZM and fZM as LNRE models

- Discrete Zipf-Mandelbrot population

$$\pi_i := \frac{C}{(i+b)^a} \quad \text{for } i = 1, \ldots, S$$

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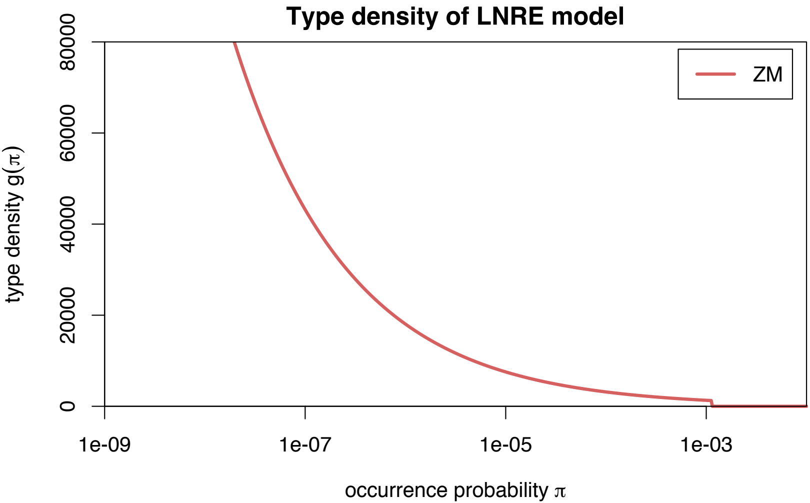
with parameters
  - $\alpha = 1/a$ $(0 < \alpha < 1)$
  - $B = b \cdot \alpha/(1-\alpha)$
  - $0 \leq A < B$ determines S (ZM with $S = \infty$ for $A = 0$)

☞ C is a normalization factor, not a parameter

ZM and fZM as LNRE models



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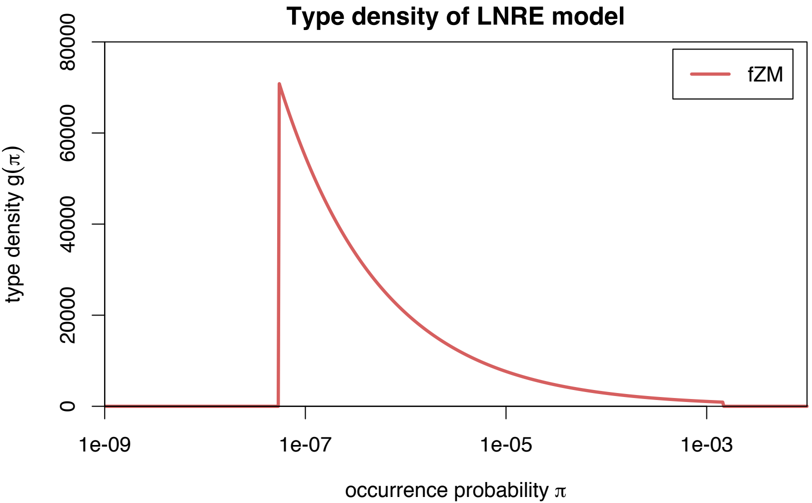

Expectations as integrals

- Expected values can now be expressed as integrals over $g(\pi)$

$$\mathrm{E}[V_m] = \int_0^\infty \frac{(N\pi)^m}{m!} e^{-N\pi} g(\pi)\, d\pi$$

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$$\mathrm{E}[V] = \int_0^\infty (1 - e^{-N\pi}) g(\pi)\, d\pi$$

- Reduce to simple closed form for ZM (approximation)

$$\mathrm{E}[V_m] = \frac{C}{m!} \cdot N^\alpha \cdot \Gamma(m - \alpha)$$
$$\mathrm{E}[V] = C \cdot N^\alpha \cdot \frac{\Gamma(1 - \alpha)}{\alpha}$$

- fZM and exact solution for ZM with incompl. Gamma function

Parameter estimation from training corpus

- For ZM, $\alpha = \frac{\mathrm{E}[V_1]}{\mathrm{E}[V]} \approx \frac{V_1}{V}$ can be estimated directly, but prone to overfitting
- General parameter fitting by **MLE**: maximize likelihood of observed spectrum **v**

$$\max_{\alpha,A,B} \Pr\big((V, V1, \ldots, V_k) = \mathbf{v} \,\big|\, \alpha, A, B\big)$$

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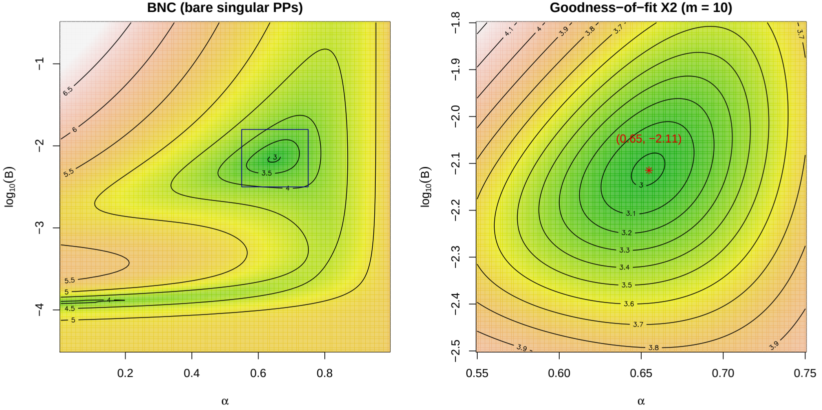
- Multivariate normal approximation:

$$\min_{\alpha,A,B} (\mathbf{v} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{v} - \boldsymbol{\mu})$$

- Minimization by gradient descent (BFGS, CG) or simplex search (Nelder-Mead)

Parameter estimation from training corpus



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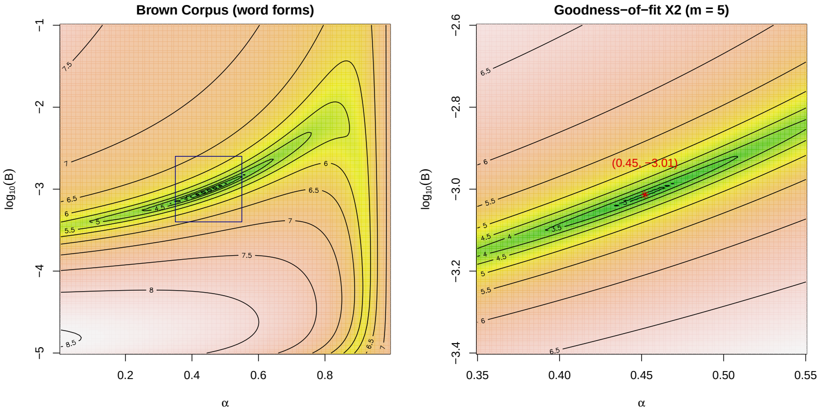

Goodness-of-fit

(Baayen 2001, Sec. 3.3)

- How well does the fitted model explain the observed data?
- For multivariate normal distribution:

$$X^2 = (\mathbf{V} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{V} - \boldsymbol{\mu}) \sim \chi^2_{k+1}$$

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Goodness-of-fit

(Baayen 2001, Sec. 3.3)

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➥ Multivariate chi-squared test of **goodness-of-fit**

- replace $\mathbf{V}$ by observed $\mathbf{v}$ ➜ test statistic x^2
- must reduce df $= k + 1$ by number of estimated parameters

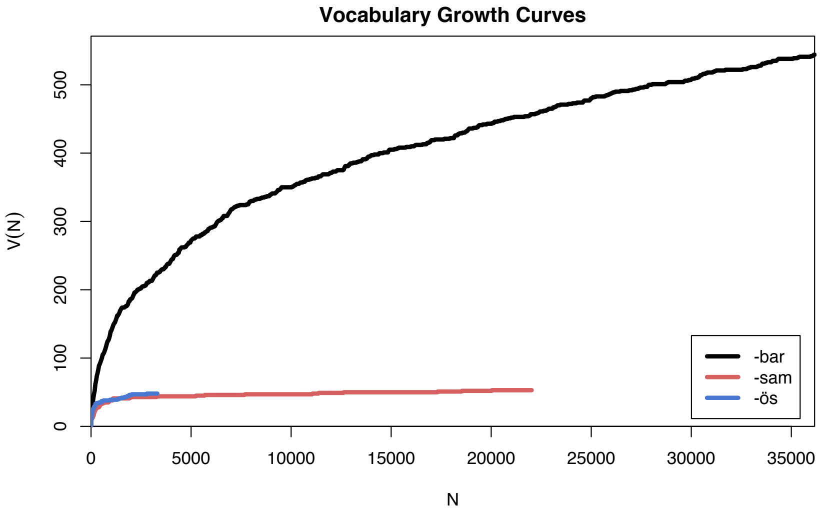
- NB: significant rejection of the LNRE model for $p < .05$

Coffee break!

Outline

Measuring morphological productivity

example from Evert and Lüdeling (2001)



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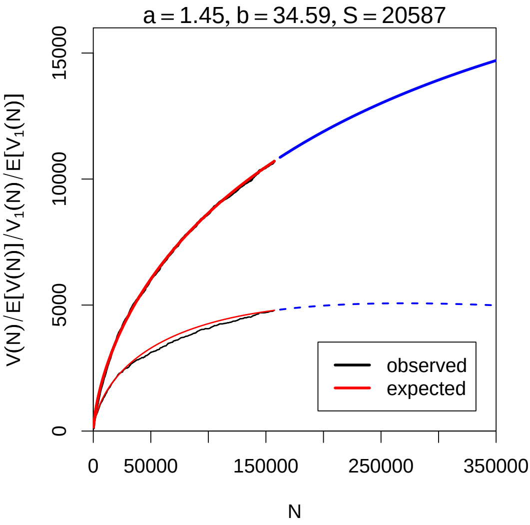

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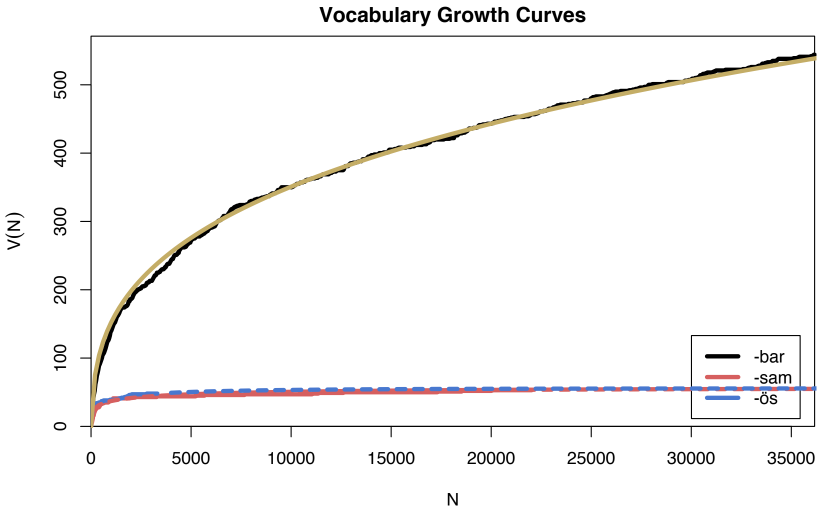

Quantitative measures of productivity

(Tweedie and Baayen 1998; Baayen 2001)

- Baayen's (1991) productivity index $\mathcal{P}$ (slope of vocabulary growth curve)

$$\mathcal{P} = \frac{V_1}{N}$$

- TTR = type-token ratio

$$\mathrm{TTR} = \frac{V}{N}$$

- Zipf-Mandelbrot slope

$$a$$

- Herdan's law (1964)

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$$C = \frac{\log V}{\log N}$$

- Yule (1944) / Simpson (1949)

$$K = 10\,000 \cdot \frac{\sum_m m^2 V_m - N}{N^2}$$

- Guiraud (1954)

$$R = \frac{V}{\sqrt{N}}$$

- Sichel (1975)

$$S = \frac{V_2}{V}$$

- Honoré (1979)

$$H = \frac{\log N}{1 - \frac{V_1}{V}}$$

Productivity measures for bare singulars in the BNC

	spoken	written
V	2,039	12,876
N	6,766	85,750
K	86.84	28.57
R	24.79	43.97
S	0.13	0.15
C	0.86	0.83
$\mathcal{P}$	0.21	0.08
TTR	0.301	0.150
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pop. S	15,958	36,874

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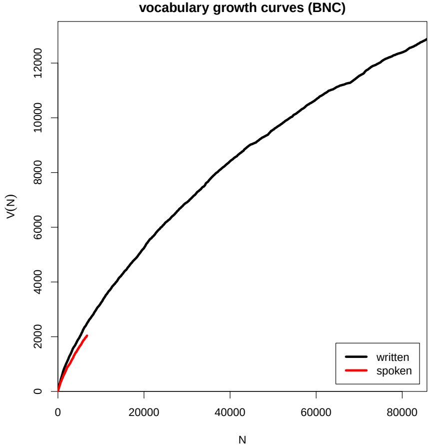
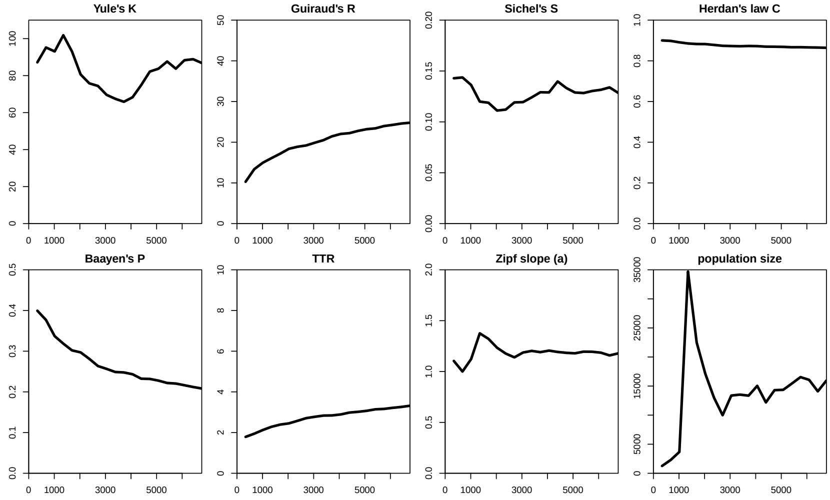


Are these "lexical constants" really constant?

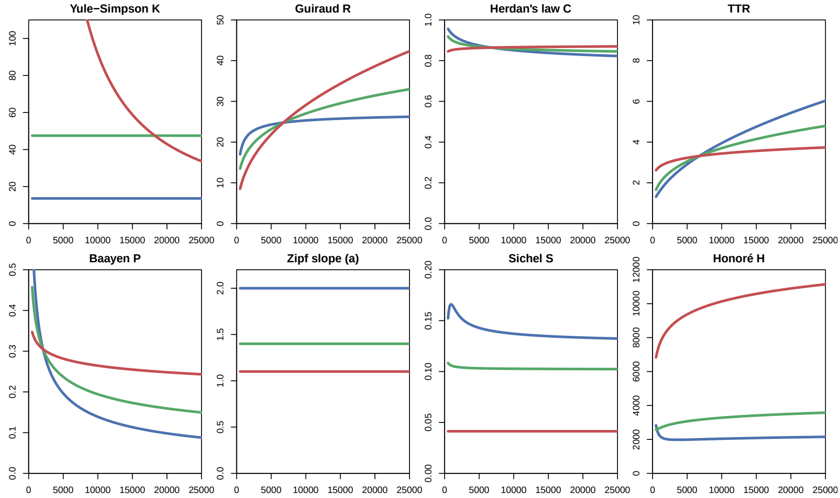


Simulation experiments based on LNRE models

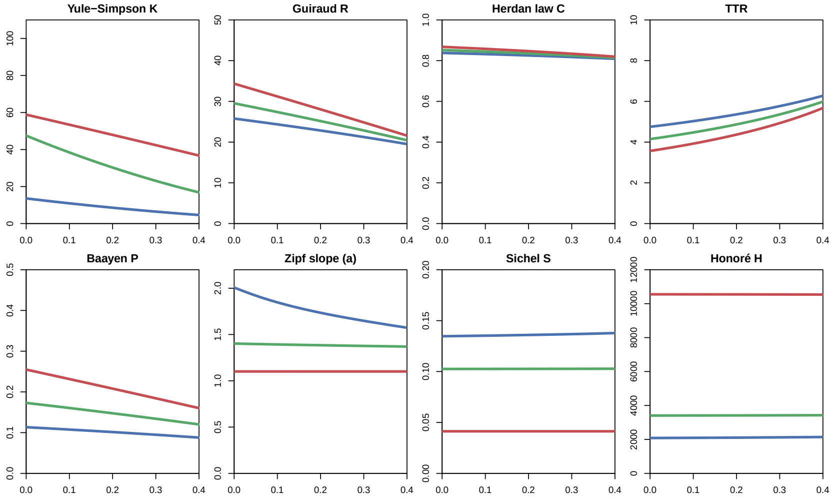
- Systematic study of size dependence and other aspects of productivity measures based on samples from LNRE model
- LNRE model ➜ well-defined population
- Random sampling helps to assess variability of measures
- Expected values $\mathrm{E}[\mathcal{P}]$ etc. can often be computed directly (or approximated) ➜ computationally efficient

➥ LNRE models as tools for understanding productivity measures

Simulation: sample size

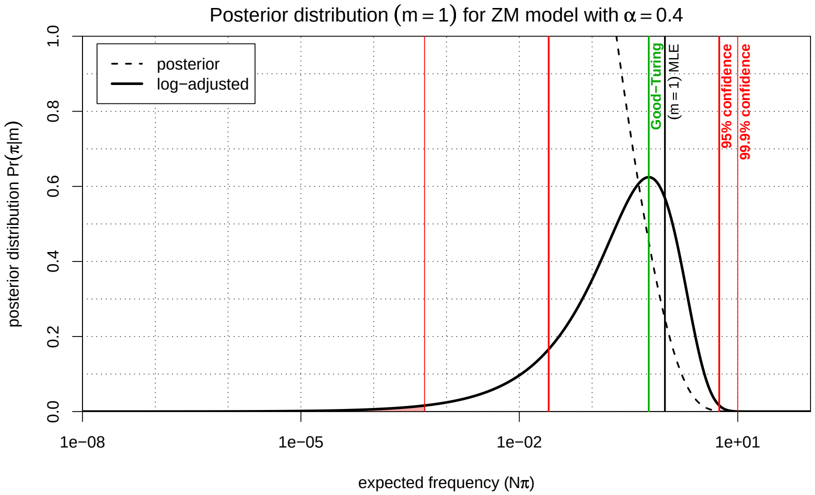


Simulation: frequent lexicalized types

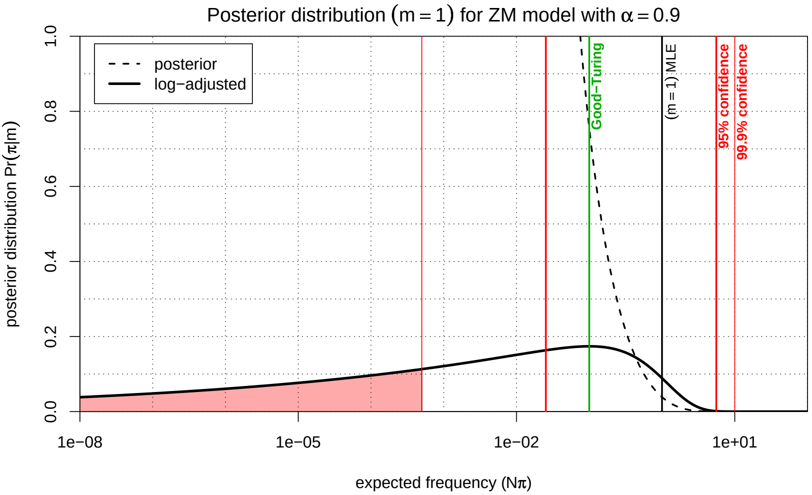


interactive demo

Posterior distribution



Posterior distribution



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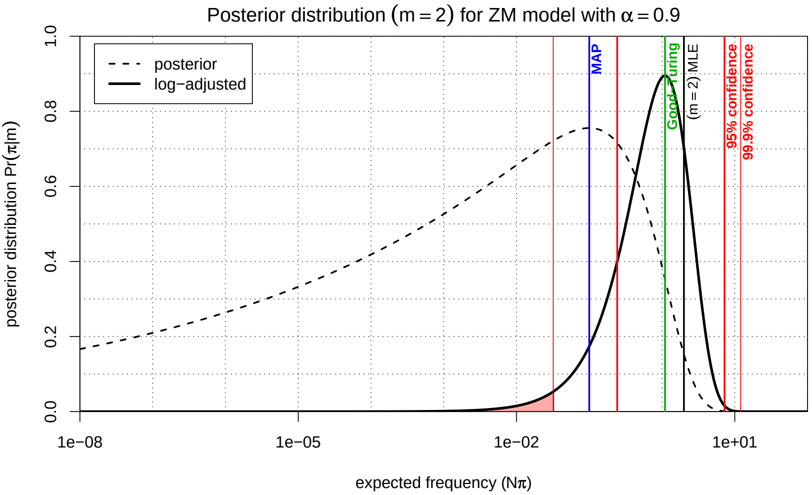

Outline

How reliable are the fitted models?

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Bootstrapping

- An empirical approach to sampling variation:
  - take many random samples from the same population
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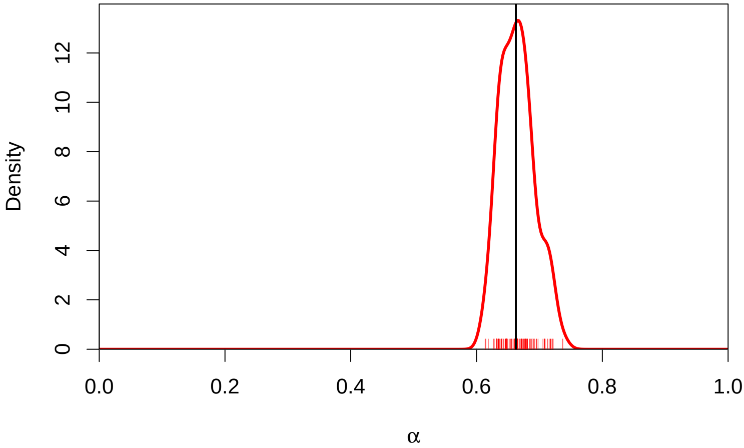
- Parametric bootstrapping
  - use fitted model to generate samples, i.e. sample from the population described by the model
  - advantage: "correct" parameter values are known

Bootstrapping

parametric bootstrapping with 100 replicates

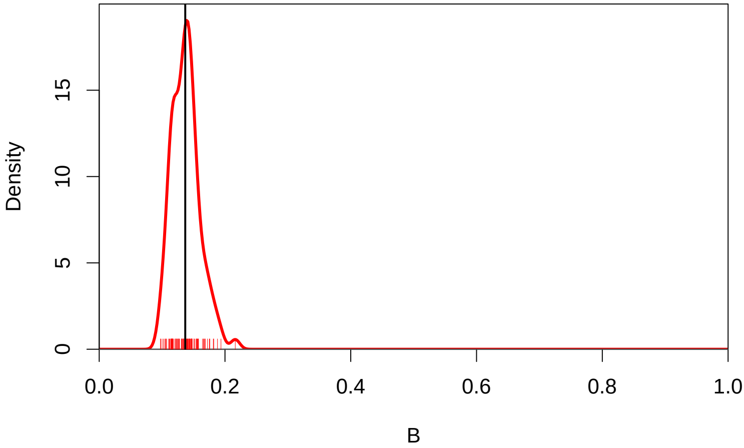
Zipfian slope $a = 1/\alpha$



Bootstrapping

parametric bootstrapping with 100 replicates

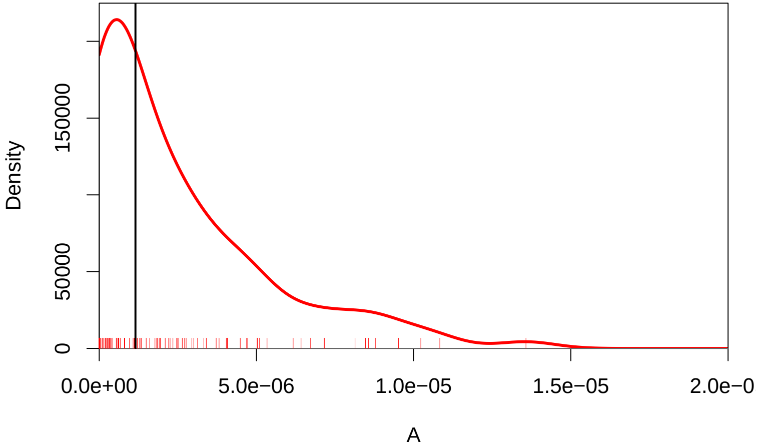
Offset $b = (1 - \alpha)/(B \cdot \alpha)$



Bootstrapping

parametric bootstrapping with 100 replicates

fZM probability cutoff $A = \pi_S$



Bootstrapping

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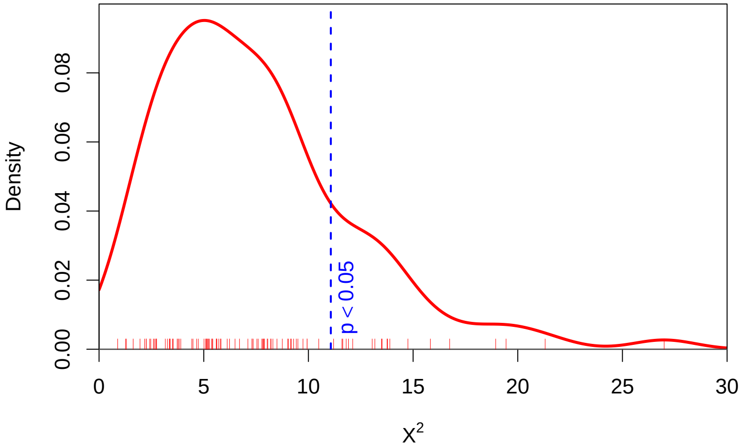
Goodness-of-fit statistic X^2 (model not plausible for $X^2 > 11$)



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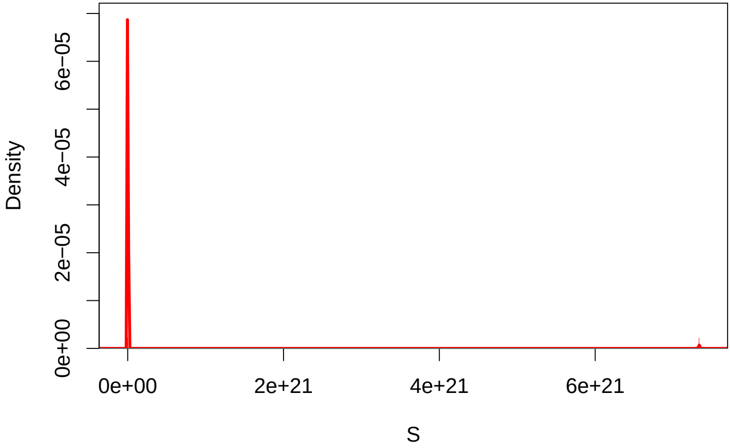
Population diversity S



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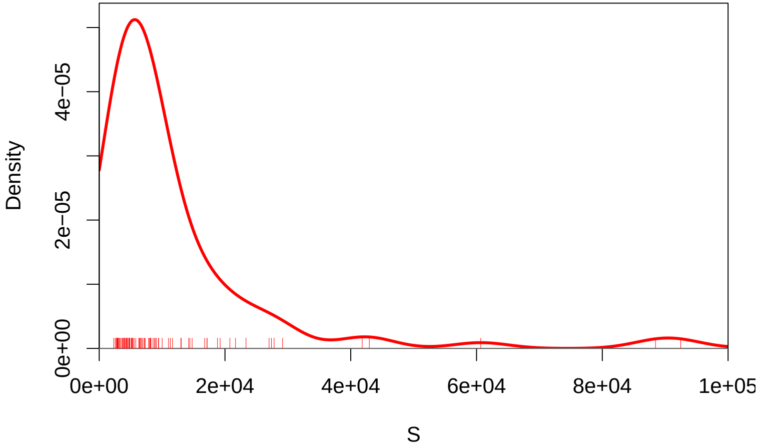
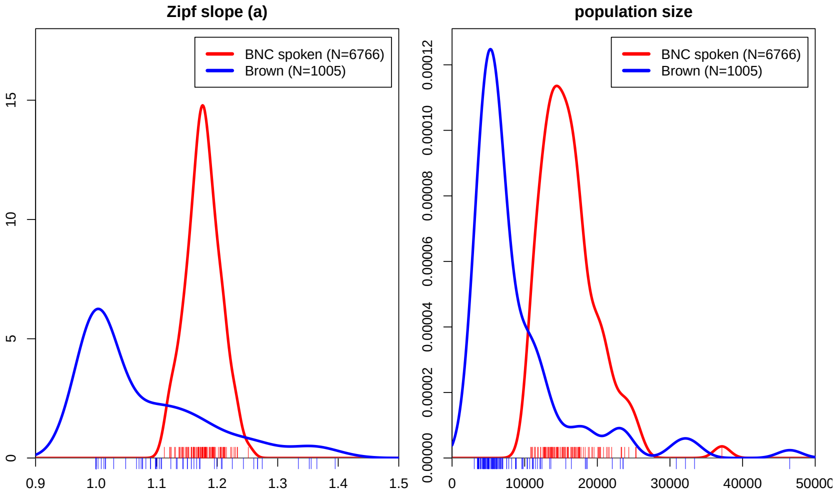


Sample size matters!

Brown corpus is too small for reliable LNRE parameter estimation (bare singulars)



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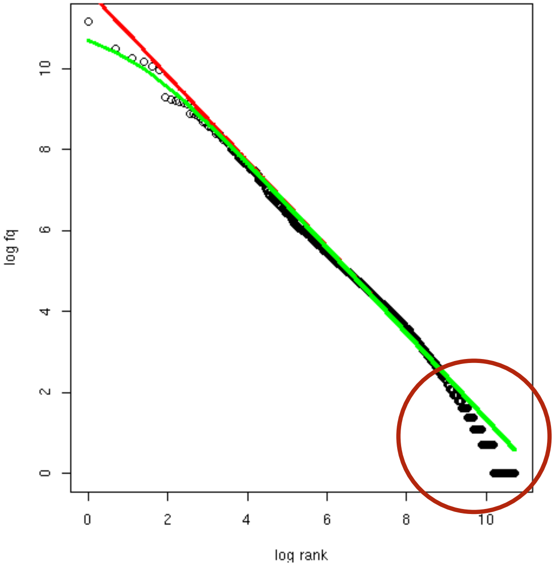
How well does Zipf's law hold?



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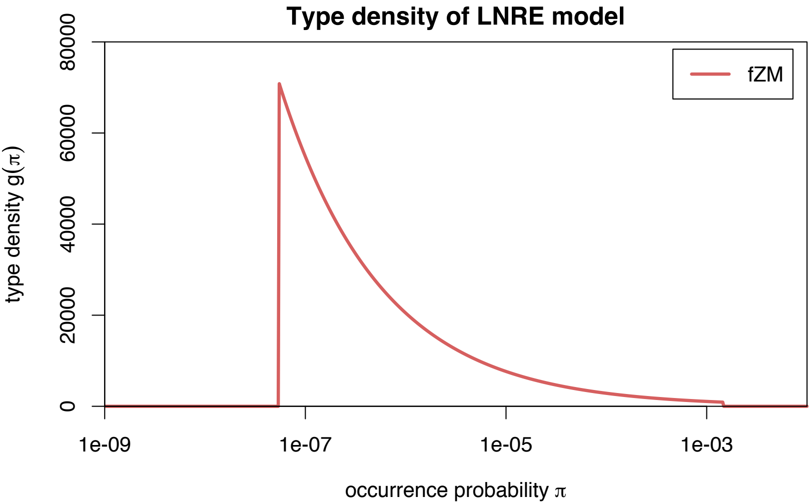
- E.g. Generalized Inverse Gauss-Poisson (GIGP; Sichel 1971)

$$g(\pi) = \frac{(2/bc)^{\gamma+1}}{K_{\gamma+1}(b)} \cdot \pi^{\gamma-1} \cdot e^{-\frac{\pi}{c} - \frac{b^2 c}{4\pi}}$$

The GIGP model (Sichel 1971)



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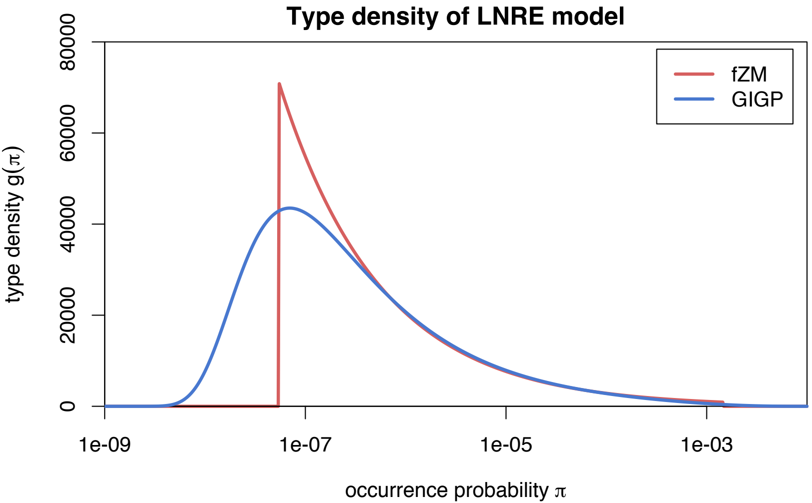
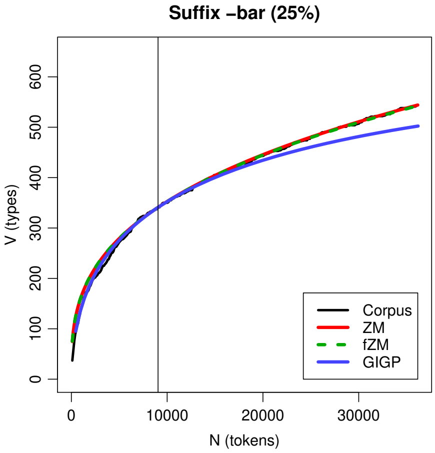
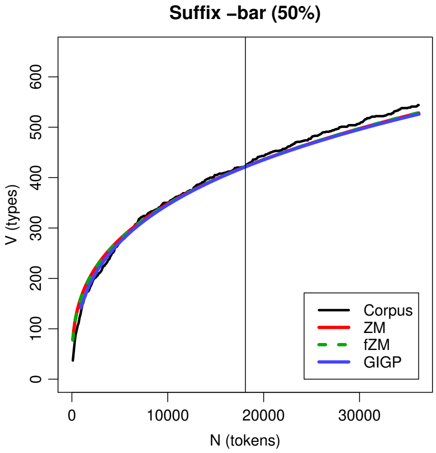


Outline

How accurate is LNRE-based extrapolation?

(Baroni and Evert 2005)





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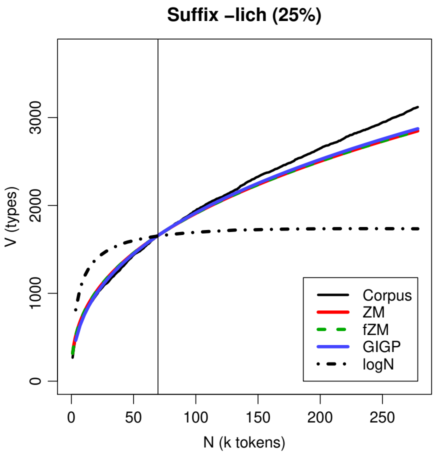

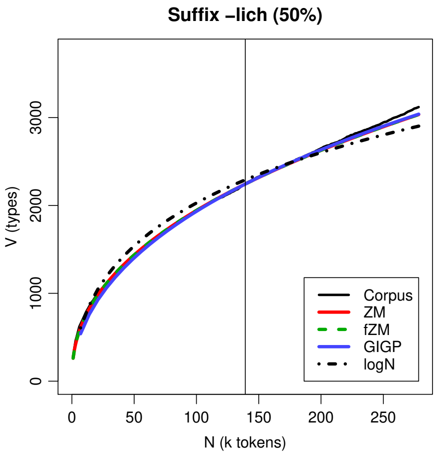

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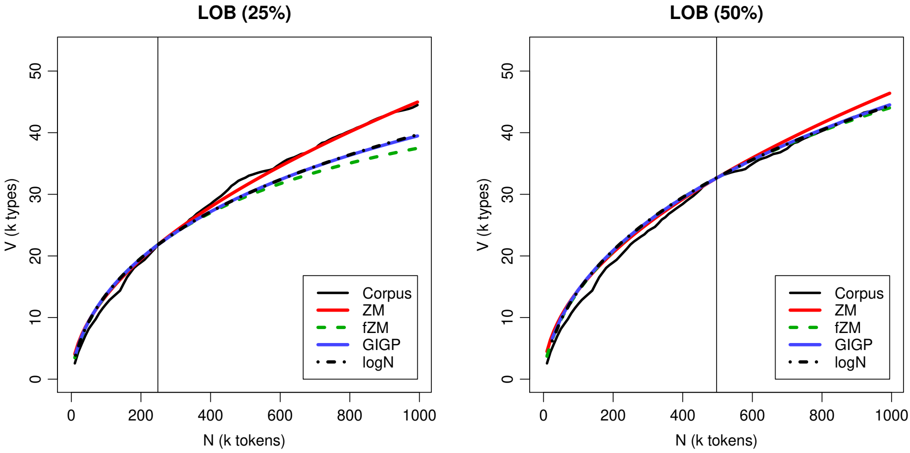

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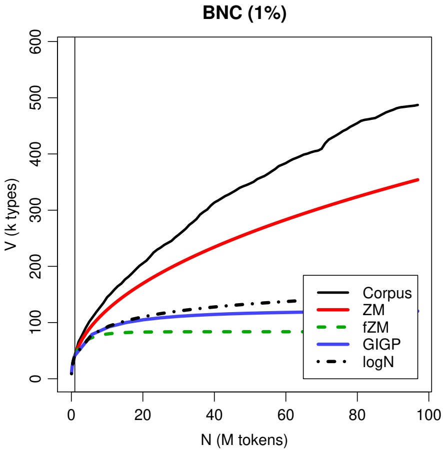

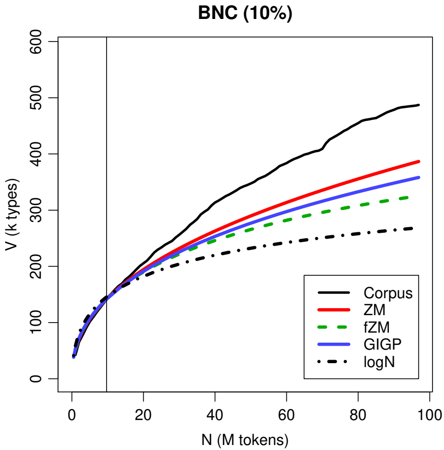

Reasons for poor extrapolation quality

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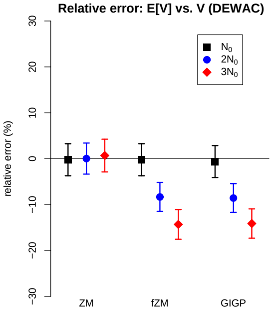
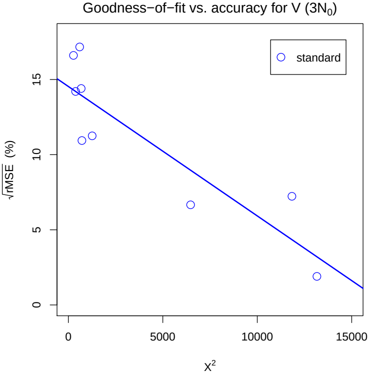
  - well-known in computational linguistics (Church 2000)
- Cause 2: **non-homogeneous** corpus
  - cannot extrapolate from spoken BNC to written BNC
  - similar for different genres and domains
  - also within single text, e.g. beginning/end of novel

The ECHO correction

(Baroni and Evert 2007)

- Empirical study: quality of extrapolation $N_0 \rightarrow 4N_0$ starting from random samples of corpus texts





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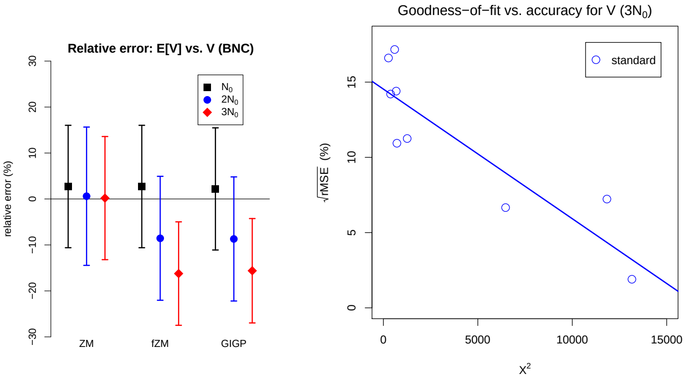

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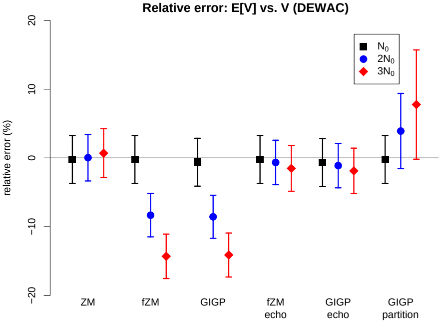

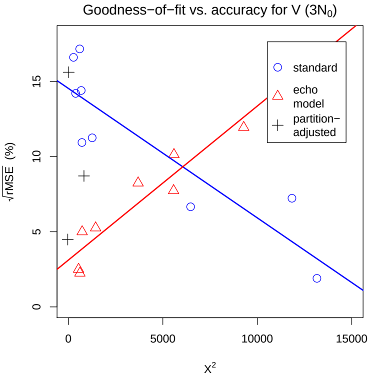

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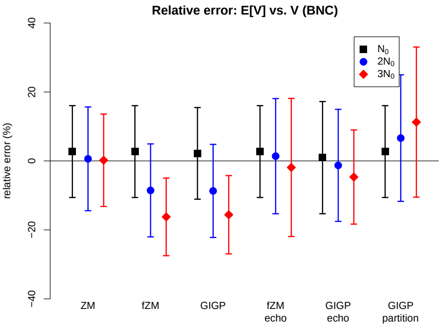

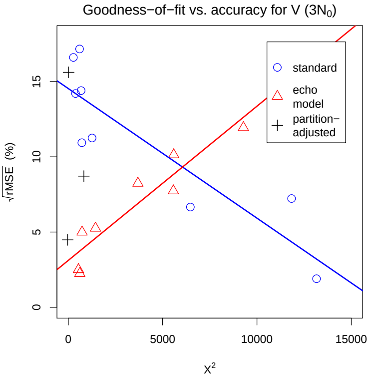

Outline

Future plans for zipfR

- More efficient LNRE sampling & parametric bootstrapping
- Improve parameter estimation (minimization algorithm)
- Better computation accuracy by numerical integration
- Extended Zipf-Mandelbrot LNRE model: piecewise power law
- Development of robust and interpretable productivity measures, using LNRE simulations
- Computationally expensive modelling (MCMC) for accurate inference from small samples

Thank you!

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